Joint Channel and Carrier Offset Estimation in CDMA Communications

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***Abstract*— This paper addresses the problem of CDMA multiuser detection in the presence of unknown multipath channels and residual carriers. An analytic algorithm that provides closed-form channel and carrier offset estimates is proposed. The algorithm first converts the multiuser estimation problem into single-user problems and then analytically solves the resulting nonlinear multivariate optimization problems using a polynomial matrix projection property. The performance of the proposed algorithm is investigated through first-order perturbation analysis. We also calculate the Cramér–Rao bound (CRB) to illustrate the efficiency of the new algorithm. The analyzes are supported by computer simulations.**

***Index Terms*— Blind identification, CDMA, channel equalization and carrier synchronization.**

I. Introduction

THE PROBLEM considered in this paper is that of signal recovery in code division multiple access (CDMA) communications with unknown multipath channels and carrier offsets. It is well known that in direct-sequence CDMA, multiuser detection can be performed to decouple the received signals using knowledge of users' signature waveforms [1], [2]. In practice, however, the source separation problem is complicated by the fact that users' signature waveforms may be distorted by unknown channel effects and carrier offsets. The everchanging multipath channels and the inevitable residual carriers[1] present great technical challenges in high speed CDMA communications.

Blind techniques have been proposed to estimate the multipath channels and carrier offsets at the receiver. When used properly in wireless communications, they can provide significant improvement in system capacity and robustness against environmental variations. Although without a pilot signal both the multipath channels and the carrier offsets must be estimated, carrier synchronization and channel identification have traditionally been treated as separate problems. The later has been well studied in the context of *blind equalization* (e.g., [3]–[6]) for single user systems. Most research in this area, however, implicitly assumes that the carrier offset has been properly compensated. Similarly many existing carrier recovery techniques consider only flat channels and intersymbol interference (ISI) free samples. Joint treatment of multipath channels and carrier offsets has shown to be advantageous; a Viterbi type joint blind sequence estimation and carrier recovery approach was proposed in [7]. Only limited studies on joint estimation are available, almost all of which are on single-user systems [8]. Furthermore, most of the solutions developed to date rely on iterative searching due to the nonlinear nature of the problem.

Assuming perfect carrier synchronization, the authors presented in [9] and [10] a subspace-based channel estimation algorithm that can accomplish signature waveform estimation in CDMA without pilot signals. Similar techniques have been developed in [11] and [12]. Other results on blind source separation can be found in [13]–[15]. To cope with the imperfect carrier recovery, an ESPRIT-based algorithm has been proposed for carrier offset estimation in a multiuser setup [16]. However, association of the estimated carriers remains a much involved problem. Note that unlike a single-user system, in which the channel and carrier offset can be dealt with separately, joint estimation is compulsory in CDMA systems due to the presence of multiuser interference. Although, in principle, the joint estimation problem can be solved using the maximum likelihood (ML) method, the complexity associated with exceedingly high dimensional optimization renders the very idea impractical.

In this paper, we seek an analytic solution for the estimation of unknown multipath channels and carrier offsets parameters in CDMA. Built on the algorithm in [10], the proposed method first transforms the joint multiuser estimation problem into a set of single-user multiparameter estimation problems. Each single-user problem is then reduced and solved by a simple polynomial rooting or one-dimensional (1-D) spectrum searching through certain polynomial matrix operation. The new approach avoids the convergence and local minima issues common to adaptive approaches and offers closed-form estimation of channels and carrier offsets for all users utilizing standard numerical tools. Performance of the proposed estimator is studied using perturbation analysis and is verified by computer simulations. The efficacy of the proposed algorithm is demonstrated by comparing its performance with the Cramér–Rao bound (CRB) derived in this paper.

The rest of this paper is organized as follows. In Section II, a base-band discrete-time CDMA model that accounts for both

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[1] When multiple signals are present, their carriers will differ, in general, due to independent oscillators.

the multipath channel and carrier offset effects is formulated. The proposed algorithm is presented in Section III, along with some discussions on symbol recovery. Section IV provides performance analysis and the derivation of the CRB. Several illustrative computer simulations are given in Section V. The paper is then concluded in Section VI.

Notations used in this paper are standard. Symbols for matrices (in capital letter) and vectors (lower case) are in boldface. $(\cdot)^T, (\cdot)^H, (\cdot)^*, (\cdot)^\dagger$, and $\otimes$ denote transpose, Hermitian, conjugate, pseudo-inverse, and linear convolution respectively. The symbol $\boldsymbol{I}(\boldsymbol{0})$ stands for the identity (zero) matrix with proper dimension. $\hat{\theta}$ denotes estimate of parameter θ, and $\|\cdot\|$ denotes the 2-norm.

II. Data Formulation

In a typical wireless scenario, the received noise free baseband signal is given by

$$y(t) = \sum_{l=0}^{L_c-1} \alpha_l x(t-\tau_l) e^{j\omega(t-\tau_l)} \tag{1}$$

where

- $x(t)$ baseband transmitted signal;
- L_c number of multipath reflections;
- τ_l delay associated with the lth multipath;
- α_l complex attenuation associated with the lth multipath;
- ω residual carrier due to imperfect carrier synchronization.

For a linearly modulated signal, $x(t) = \Sigma_{n=-\infty}^{\infty} b(n)g(t-nT)$, where

- $b(n)$ bit sequence;
- $g(t)$ pulse-shaping function;
- T bit period.

On defining $h(t) = \Sigma_{l=0}^{L_c-1} \alpha_l g(t-\tau_l)e^{-j\omega\tau_l}$ as the composite channel response, (1) can be simplified as

$$y(t) = \sum_{n=-\infty}^{\infty} b(n)h(t-nT)e^{j\omega t}. \tag{2}$$

The bit-rate discrete-time equivalent of the above model is thus given by

$$y(k) = \sum_{n=-\infty}^{\infty} b(n)h(k-n)e^{j\phi k} \tag{3}$$

where $y(k) = y(t)|_{t=kT}$, $h(k) = h(t)|_{t=kT}$, and $\phi = \omega T$. Here, $h(k)$ and ϕ typify the two types of impairments commonly encountered in a wireless environment, namely, the multipath channel and the carrier offset. It is in general plausible to model $h(k)$ as an FIR filter with maximum length L [2].

In a CDMA system with P users, the superimposed received signal can be modeled as

$$\begin{aligned} y(k) &= \sum_{i=1}^{P} y_i(k) + n(k) \\ &= \sum_{i=1}^{P} \sum_{n=-\infty}^{\infty} b_i(n)e^{j\phi_i k}h_i(n-k) + n(k) \end{aligned} \tag{4}$$

where

- i user index;
- $y_i(k)$ ith user's received signal;
- $n(k)$ additive noise.

The signal from each user is modulated by a distinct spreading sequence. Let M be the spreading factor and $c_i(n), n = 0, \cdots, M-1$ be the spreading code for the ith user; then, the chip sequence in CDMA can be expressed as

$$b_i(k) = \sum_{l=-\infty}^{\infty} s_i(l)c_i(k-lM) \tag{5}$$

or equivalently

$$b_i(kM+m) = s_i(k)c_i(m), m = 0, \cdots, M-1.$$

Here $s_i(l)$ is the ith user's information bearing symbol stream. Clearly, the symbol period T_s is M times the chip period or sample interval T.

In this paper, we consider quasisynchronous CDMA systems, where signals from all users are synchronized within L chip durations. More specifically, we assume that the timing ambiguity is accounted for by the unknown channels of length L. Since $L \ll M$ for most CDMA applications [1], the channel effect will cause minor ISI ($L-1$ out of every M samples). Out of the M chip-rate samples within the kth symbol period, the majority $(M-L+1)$ samples are ISI free and are given by

$$\begin{aligned} &y(kM+m) \\ &\quad = \sum_{i=1}^{P} s_i(k)e^{jkM\phi_i} \sum_{l=0}^{L-1} e^{jm\phi_i}c_i(m-l)h_i(l) \\ &\qquad + n(kM+m), \qquad L-1 \le m < M-1. \end{aligned} \tag{6}$$

For analytical simplicity, we shall only consider these ISI free samples in each symbol period in the remainder of this paper.[2] It should be pointed out that the algorithm to be presented here can be extended to asynchronous CDMA system with minor modifications.

On defining $w_i(n) = \Sigma_{l=0}^{L-1} e^{j\phi_i n}c_i(n-l)h_i(l)$ and $K = M-L+1$, we may rewrite these ISI-free samples within the kth symbol period in vector form as

$$\begin{aligned} \boldsymbol{y}(k) &= \begin{bmatrix} y(kM+L-1) \\ y(kM+L) \\ \vdots \\ y((k+1)M-1) \end{bmatrix} \\ &= \sum_{i=1}^{P} s_i(k)e^{jkM\phi_i} \begin{bmatrix} w_i(L-1) \\ w_i(L) \\ \vdots \\ w_i(M-1) \end{bmatrix} + \begin{bmatrix} n(kM+L-1) \\ n(kM+L) \\ \vdots \\ n((k+1)M-1) \end{bmatrix} \\ &\overset{\text{def}}{=} \sum_{i=1}^{P} s_i(k)e^{jkM\phi_i}\boldsymbol{w}_i + \boldsymbol{n}(k). \end{aligned} \tag{7}$$

[2] Alternatively we may utilize all samples by introducing guard time in spreading code, i.e., by setting $c_i(m) = 0$, for $m = M-L, \cdots, M-1$, to avoid ISI.

It is easy to verify that $\boldsymbol{w}_i$ can be expressed as

$$\boldsymbol{w}_i = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{j\phi_i} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\phi_i(K-1)} \end{bmatrix}}_{\boldsymbol{Z}_i} \cdot \underbrace{\begin{bmatrix} c_i(L-1) & \cdots & c_i(0) \\ c_i(L) & \cdots & c_i(1) \\ \vdots & \ddots & \vdots \\ c_i(M-1) & \cdots & c_i(K-1) \end{bmatrix}}_{\boldsymbol{C}_i} \underbrace{\begin{bmatrix} h_i(0) \\ \vdots \\ h_i(L-1) \end{bmatrix}}_{\boldsymbol{h}_i} = \boldsymbol{Z}_i \boldsymbol{C}_i \boldsymbol{h}_i. \tag{8}$$

The general problem addressed in this paper is the estimation of $\{\boldsymbol{w}_i\}$ or $\{\phi_i\}$ and $\{\boldsymbol{h}_i\}$ from $\{\boldsymbol{y}(k)\}_{k=1}^N$ without the knowledge of information symbol $\{s_i(k)\}$.

As will become clear later, the algebraic structure revealed in $\boldsymbol{w}_i = \boldsymbol{Z}_i\boldsymbol{C}_i\boldsymbol{h}_i$ plays an important role in solving the joint estimation problem. Once $\{\boldsymbol{w}_i\}_{i=1}^P$ are estimated, multiuser detection can easily be performed using decorrelating or MMSE receivers [17]–[21].

III. The Proposed Algorithm

Equations (7) and (8) present a well-defined parameter estimation problem. Theoretically the problem can be tackled using the maximum likelihood estimator (MLE) [22]. The parameter set can be determined by maximizing the likelihood function. This, however, generally involves multidimensional searching, which has some well-known implementational difficulties. In the following, we seek a more practical approach with lower complexity and maybe suboptimal performance.

A. Joint Estimation

Our proposed approach accomplishes closed-form parameter estimation in two steps: First, it reduces the multiuser problem into P single-user estimation problems through subspace decomposition; then, it simplifies each single-user problem to a tractable 1-D minimization problem. Key to the present algorithm is a polynomial matrix operation that enables us to decouple carrier and channel in the cost function.

To simplify our presentation, we will study noise-free case and rewrite (7) as

$$\boldsymbol{y}(k) = \sum_{i=1}^{P} s_i(k)e^{jkM\phi_i}\boldsymbol{w}_i = \underbrace{[\boldsymbol{w}_1 \cdots \boldsymbol{w}_P]}_{\boldsymbol{W}} \begin{bmatrix} s_1(k)e^{jkM\phi_1} \\ \vdots \\ s_P(k)e^{jkM\phi_P} \end{bmatrix}. \tag{9}$$

We first collect N output sample vectors and obtain

$$\boldsymbol{Y} = [\boldsymbol{y}(1)\cdots\boldsymbol{y}(N)] = [\boldsymbol{w}_1\cdots\boldsymbol{w}_P] \cdot \begin{bmatrix} s_1(1) & s_1(2)e^{jM\phi_1} & \cdots & s_1(N)e^{j(N-1)M\phi_1} \\ \vdots & & \ddots & \vdots \\ s_P(1) & s_P(2)e^{jM\phi_P} & \cdots & s_P(N)e^{j(N-1)M\phi_P} \end{bmatrix} \stackrel{\text{def}}{=} \boldsymbol{WS}. \tag{10}$$

A singular value decomposition (SVD) can be performed on $\boldsymbol{Y}$ as

$$\boldsymbol{Y} = \boldsymbol{WS} = (\boldsymbol{U}_s \quad \boldsymbol{U}_o)\begin{pmatrix} \Sigma_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}\begin{pmatrix} \boldsymbol{V}_s^H \\ \boldsymbol{V}_o^H \end{pmatrix}. \tag{11}$$

With sufficient data samples, it is plausible to assume the signal matrix $\boldsymbol{S}$ is of full row rank.[3] Then, $\boldsymbol{U}_s$ and $\boldsymbol{W}$ share the same column subspace, and $\boldsymbol{U}_o$ is the orthogonal complement. From (8) and (11), the homogeneous equations

$$\boldsymbol{U}_o \perp \boldsymbol{W} \Rightarrow \boldsymbol{U}_o^H\boldsymbol{w}_i = \boldsymbol{U}_o^H\boldsymbol{Z}_i\boldsymbol{C}_i\boldsymbol{h}_i = \boldsymbol{0}, \qquad i = 1,\cdots,P. \tag{12}$$

hold. Equation (12) reduces the multiuser parameter estimation to P single-user problems. On defining $J(\phi_i, \boldsymbol{h}_i) = \|\boldsymbol{U}_o^H\boldsymbol{Z}_i\boldsymbol{C}_i\boldsymbol{h}_i\|^2$, we can estimate the carrier and channel of each user as

$$\hat{\phi}_i, \hat{\boldsymbol{h}}_i = \underset{\phi_i, \boldsymbol{h}_i}{\operatorname{argmin}} \ \|J(\phi_i, \boldsymbol{h}_i)\|^2 \qquad i = 1,\cdots,P. \tag{13}$$

Relative to the MLE, the complexity associated with the above problem is significantly lower. However, (13) is still a nontrivial multidimensional minimization problem because of the nonlinear parameters involved. A simpler case with zero carrier offset has been studied in [10]. In this case, $\boldsymbol{Z}_i$ in (12) reduces to an identity matrix. Consequently, (13) has a quadratic form and can be solved using standard minimization methods

$$\hat{\boldsymbol{h}}_i = \underset{\|\boldsymbol{h}_i\|=1}{\operatorname{argmin}} \ \boldsymbol{h}_i^H\boldsymbol{C}_i^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{C}_i\boldsymbol{h}_i. \tag{14}$$

The difficulty of the present problem is two fold.

1) The estimation of ϕ_i and $\boldsymbol{h}_i$ cannot be easily decoupled.
2) $\boldsymbol{Z}_i(\phi_i) = \operatorname{diag}[1, e^{j\phi_i}, \cdots, e^{\phi_i(M-1))}]$ is nonlinearly dependent on ϕ_i.

To obtain an analytic solution, we need to convert $J(\phi, \boldsymbol{h})$ into a more tractable form.

For clarity, we drop the subscript in (12) and consider

$$\boldsymbol{U}_o^H\boldsymbol{ZCh} = \boldsymbol{0}. \tag{15}$$

Defining $z = e^{j\phi}$, (15) can be partitioned as

$$[\boldsymbol{u}_{o1} \quad \boldsymbol{u}_{o2} \quad \cdots \quad \boldsymbol{u}_{oK}]\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & z & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & z^{K-1} \end{bmatrix}\begin{bmatrix} \boldsymbol{c}_1^T \\ \boldsymbol{c}_2^T \\ \vdots \\ \boldsymbol{c}_K^T \end{bmatrix}\boldsymbol{h} = \boldsymbol{0} \tag{16}$$

where $\boldsymbol{u}_{oi}$ is the ith column vector of $\boldsymbol{U}_o^H$, and $\boldsymbol{c}_j^T$ is the jth row vector of $\boldsymbol{C}$. Note that the product $\boldsymbol{U}_o^H\boldsymbol{ZC}$ in (16) can be further expressed as a polynomial matrix with variable z

$$[\boldsymbol{u}_{o1} \quad \boldsymbol{u}_{o2} \quad \cdots \quad \boldsymbol{u}_{oK}]\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & z & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & z^{K-1} \end{bmatrix}\begin{bmatrix} \boldsymbol{c}_1^T \\ \boldsymbol{c}_2^T \\ \vdots \\ \boldsymbol{c}_K^T \end{bmatrix} = \underbrace{\sum_{i=1}^{K}\boldsymbol{u}_{oi}\boldsymbol{c}_i^T z^{i-1}}_{\boldsymbol{Q}(z)}. \tag{17}$$

[3] An equivalent assumption known as persistent excitation commonly incurs in system identification literature, which generally holds true in practice.

$\boldsymbol{Q}(z)$ is an $L \times L$ polynomial matrix of order $K-1$. Now, we can rewrite (15) as

$$\boldsymbol{Q}(z)\boldsymbol{h} = [\boldsymbol{q}_1(z), \quad \cdots, \quad \boldsymbol{q}_L(z)]\boldsymbol{h} = \boldsymbol{0}. \tag{18}$$

Clearly, $\boldsymbol{h}$ is a null vector of $\boldsymbol{Q}(z)$ when $z_0 = e^{j\phi_0}$, where ϕ_0 is true carrier offset. In other words, the nullity of $\boldsymbol{Q}(e^{j\phi_0})$ is at least 1. Barring the degeneration case, this suggests that ϕ_0 can be determined by examining the rank condition of $\boldsymbol{Q}(z)$ along the unit circle. For z and $\boldsymbol{h}$ to be uniquely identifiable, it is necessary for $\boldsymbol{Q}(z)$ to have full rank, except for $z = z_0$. That is, one diagonal element of the Smith form of $\boldsymbol{Q}(z)$ must have a zero at z_0, whereas all other diagonal elements are constant. The exact identifiability conditions and their physical interpretation are currently under investigation.

To evaluate the nullity of $\boldsymbol{Q}(z)$, let

$$\boldsymbol{F}(z) = [\boldsymbol{q}_2(z), \cdots, \boldsymbol{q}_L(z)] \tag{19}$$

and note from linear algebra that if a vector $\boldsymbol{x}$ lies in the column span of a matrix $\boldsymbol{A}$, $\boldsymbol{P}_A^{\perp}\boldsymbol{x} = 0$. Here, $\boldsymbol{P}_A^{\perp}$ is the orthogonal projection matrix of $\boldsymbol{A}$. Therefore, if we denote $\boldsymbol{P}_F^{\perp}(z)$ as the orthogonal projection polynomial matrix of $\boldsymbol{F}(z)$, ϕ_0 can be solved as

$$\phi_0 = \underset{\phi}{\operatorname{argmin}} \, \|\boldsymbol{P}_F^{\perp}(z)\boldsymbol{q}_1(z)\|. \tag{20}$$

To construct $\boldsymbol{P}_F^{\perp}(z)$, we need a finite-order polynomial matrix $\boldsymbol{G}(z)$ satisfying

$$\boldsymbol{G}(z)\boldsymbol{F}(z) = z^{-n_0}\boldsymbol{I} \tag{21}$$

where n_0 is an appropriate delay index. The product $\boldsymbol{F}(z)\boldsymbol{G}(z)$ is a valid projection matrix of $\boldsymbol{F}(z)$ with delay n_0. In Appendix A, we show how $\boldsymbol{G}(z)$ can be constructed directly from $\boldsymbol{F}(z)$. Discussions on the existence of such an FIR inverse are also provided.

Once $\boldsymbol{G}(z)$ is constructed, the orthogonal projection matrix for $\boldsymbol{F}(z)$, which is still an FIR polynomial matrix, is given by

$$\boldsymbol{P}_F^{\perp}(z) = (z^{-n_0}\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z)). \tag{22}$$

Substituting (22) into (20) and defining

$$\boldsymbol{p}(z) \stackrel{\text{def}}{=} (z^{-n_0}\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))\boldsymbol{q}_1(z) \tag{23}$$

$z_0 = e^{j\phi_0}$ is the solution of the minimization problem

$$z_0 = \underset{z}{\operatorname{argmin}} \, \|\boldsymbol{p}(z)\|^2. \tag{24}$$

Defining $a(z) = \|\boldsymbol{p}(z)\|^2 = \boldsymbol{p}^H(z)\boldsymbol{p}(z)$, ϕ_0 is readily determined by a 1-D spectrum searching of $a(z)$ along the unit circle. On the other hand, it is noticed that $a(z)$ forms a polynomial of z, which has a root $z_0 = e^{j\phi_0}$ on the unit circle. This enables us to identify ϕ_0 through polynomial rooting, as in the Root-MUSIC algorithm [23]. In practice, we pick up the root inside unit circle with the largest magnitude. Its angle is the estimate of ϕ_0.

Once $z = e^{j\phi}$ is determined, the channel $\boldsymbol{h}$ can be estimated from (12) as

$$\hat{\boldsymbol{h}} = \underset{\|\boldsymbol{h}=1\|}{\operatorname{argmin}} \, \boldsymbol{h}^H\boldsymbol{C}^H\hat{\boldsymbol{Z}}^H\boldsymbol{U}_o\boldsymbol{U}_o^H\hat{\boldsymbol{Z}}\boldsymbol{C}\boldsymbol{h} \tag{25}$$

where the channel estimate is given by the null vector of $\boldsymbol{U}_o^H\hat{\boldsymbol{Z}}\boldsymbol{C}$ corresponding to the least singular value.

The proposed algorithm is summarized below.

1) Perform an SVD on the data matrix $\boldsymbol{Y}$ to obtain the orthogonal subspace $\boldsymbol{U}_o$.
2) For each user, form the matrix polynomial $\boldsymbol{F}_i(z)$ in (19).
3) Calculate the FIR inverse $\boldsymbol{G}_i(z)$ of $\boldsymbol{F}_i(z)$ and then the orthogonal projection matrix $\boldsymbol{P}_{F_i}^{\perp}(z)$.
4) Identify the carrier offset $z_i = e^{j\phi_i}$ from cost function $a_i(z) = \|\boldsymbol{p}_i(z)\|^2$ through spectrum searching or polynomial rooting.
5) Construct $\hat{\boldsymbol{Z}}_i(z_i)$ and determine the channel vector $\boldsymbol{h}_i$ as the least-square solution of (25).

B. Multiuser Detection

Once the channel response vectors and carrier offsets are estimated, we can form a decorrelating receiver to recover each user's information bearing symbols. Note that in a multiuser scenario, the carrier offsets cannot be compensated individually. However, this does not prevent us from performing multiuser detection. First, we calculate each user's signature waveform vector using relation $\boldsymbol{w}_i = \boldsymbol{Z}_i\boldsymbol{C}_i\boldsymbol{h}_i$ and then reconstruct the $\boldsymbol{W}$ matrix in (10). To recover the ith user's signal, we design a zero-forcing receiver $\boldsymbol{f}_i$ satisfying

$$\begin{aligned}\boldsymbol{f}_i^H\boldsymbol{Y} &= \boldsymbol{f}_i^H\boldsymbol{W}\boldsymbol{S} = \boldsymbol{e}_i^H\boldsymbol{S}\\ &= [s_i(1)\ s_i(2)e^{jM\phi_i}\cdots, s_i(N)e^{j(N-1)M\phi_i}]\end{aligned} \tag{26}$$

where $\boldsymbol{e}_i$ is ith column vector of identity matrix. This is generally achievable as long as $\boldsymbol{W}$ is of full column rank.

The output from $\boldsymbol{f}_i$ is still modulated, and we can compensate the residual phase of the receiver output with the estimated ϕ_i or using differential decoding. In the noise-free case, we can perfectly recover the signals using zero-forcing receiver. With additive noise, MMSE receivers rather than zero-forcing receiver should be used as in

$$\boldsymbol{f}_i^H = \boldsymbol{R}_{yy}^{-1}\boldsymbol{w}_i^H \qquad i = 1, \cdots, P \tag{27}$$

where $\boldsymbol{R}_{yy}$ is covariance matrix of the received signals. See [17] and [18] for further discussion.

IV. PERFORMANCE ANALYSIS AND CRAMÉR–RAO BOUND

The objective of this section is to evaluate the performance of the proposed algorithm and to derive a *shape deterministic* CRB for reference. The performance measure we use here is the mean squared error (MSE) of the channel and carrier estimates.

A. Perturbation Analysis

To derive the MSE's of carrier and channel estimates of the proposed algorithm, we resort to the first-order perturbation method introduced in [24] and [25]. Since direct application of perturbation analysis to the proposed algorithm will be tedious and unnecessary, we will instead examine perturbation of the minimum point of the cost function defined in (13). This is reasonable because in constructing $\boldsymbol{p}(z)$ in (23), only matrix operations are involved. The first-order perturbation should yield close approximation for the algorithm under high SNR.

The noise corrupted data matrix can be expressed as

$$\boldsymbol{Y} = \boldsymbol{WS} + \boldsymbol{N}.$$

We first introduce a Lemma which gives an approximate expression of orthogonal subspace of noisy observations $\hat{\boldsymbol{U}}_o = \boldsymbol{U}_o + \Delta\boldsymbol{U}_o$.

Lemma 1 [24]: Let

$$\boldsymbol{x} = (\boldsymbol{U}_s \quad \boldsymbol{U}_o)\begin{pmatrix}\Sigma_s & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{V}_s^H\\ \boldsymbol{V}_o^H\end{pmatrix}$$

be the SVD of $\boldsymbol{x}$ and $\boldsymbol{N}$ be an additive noise in $\boldsymbol{Y} = \boldsymbol{x} + \boldsymbol{N}$. The first-order approximation of perturbation of $\boldsymbol{U}_o$ is given by

$$\Delta\boldsymbol{U}_o = -\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{V}_s^H\boldsymbol{N}^H\boldsymbol{U}_o = -\boldsymbol{x}^{\dagger H}\boldsymbol{N}^H\boldsymbol{U}_o. \tag{28}$$

The proposed algorithm relies on minimization of the cost function defined in (13). In the noise-free case, the cost function J will reach its minimum at the true carrier ϕ and channel $\boldsymbol{h}$

$$J(\phi, \boldsymbol{h}, \boldsymbol{U}_o) = \boldsymbol{h}^H\boldsymbol{C}^H\boldsymbol{Z}(\phi)^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{Z}(\phi)\boldsymbol{Ch} = 0. \tag{29}$$

We will abbreviate $\boldsymbol{Z}(\phi)$ as $\boldsymbol{Z}$ when no confusion is possible. In the presence of noise, J reaches its minimum at $\hat{\phi}, \hat{\boldsymbol{h}}$, and $\hat{\boldsymbol{U}}_o$. Denote $\Delta\phi = \hat{\phi} - \phi$, $\Delta\bar{\boldsymbol{h}} = \mathrm{Re}[\hat{\boldsymbol{h}} - \boldsymbol{h}]$ and $\Delta\tilde{\boldsymbol{h}} = \mathrm{Im}[\hat{\boldsymbol{h}} - \boldsymbol{h}]$. Under the high SNR assumption, $\Delta\phi$, $\Delta\bar{\boldsymbol{h}}_o$, and $\Delta\tilde{\boldsymbol{h}}_o$ can be considered very small. In the following, we shall approximate $\Delta\phi$ and $\Delta\boldsymbol{h}$ using the one-step Newton method.

In blind identification, we assume the first coefficient of $\boldsymbol{h}$ is known. This is equivalent to a linear constraint

$$\boldsymbol{e}_1^H\boldsymbol{h} = 1$$

where $\boldsymbol{e}_1$ is first column of an identity matrix with proper dimension. It should be pointed out that the actual algorithm is based on a quadratic constraint. However, within first-order, the difference between these two constraints should be small.

Denote $\boldsymbol{h}_o$ as $\boldsymbol{h}$ with the first row deleted and $\check{\boldsymbol{C}}$ as $\boldsymbol{C}$ with the first column deleted. Further, define

$$\eta = [\phi, \bar{\boldsymbol{h}}_o^T, \tilde{\boldsymbol{h}}_o^T]^T.$$

When $J(\hat{\phi}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{U}}_o)$ reaches its minimum, the first-order derivative with respect to ϕ and $\boldsymbol{h}_o$ must be 0. Approximating the first-order partial derivatives of J at $(\hat{\phi}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{U}}_o)$ by the first-order Taylor series expansion at point $(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)$ yields

$$\boldsymbol{0} = \frac{\partial J(\hat{\phi}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{U}}_o)}{\partial\eta} = \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\eta} + \frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\eta^2}\,\Delta\eta \tag{30}$$

where the second-order partial derivatives are given by

$$\frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\eta^2} = \begin{bmatrix} \frac{\partial^2 J}{\partial\phi^2} & \frac{\partial^2 J}{\partial\phi\partial\tilde{\boldsymbol{h}}_o} & \frac{\partial^2 J}{\partial\phi\tilde{\boldsymbol{h}}_o} \\ \frac{\partial^2 J}{\partial\tilde{\boldsymbol{h}}_o\partial\phi} & \frac{\partial^2 J}{\partial\tilde{\boldsymbol{h}}_o^2} & \frac{\partial^2 J}{\partial\tilde{\boldsymbol{h}}_o\partial\tilde{\boldsymbol{h}}_o} \\ \frac{\partial^2 J}{\partial\bar{\boldsymbol{h}}_o\partial\phi} & \frac{\partial^2 J}{\partial\tilde{\boldsymbol{h}}_o\partial\tilde{\boldsymbol{h}}_o} & \frac{\partial^2 J}{\partial\tilde{\boldsymbol{h}}_o^2} \end{bmatrix} \stackrel{\text{def}}{=} \boldsymbol{H}_J. \tag{31}$$

From (30), we can solve for $\Delta\phi$, $\Delta\bar{\boldsymbol{h}}$, and $\Delta\tilde{\boldsymbol{h}}$ as

$$\Delta\eta = \begin{bmatrix}\Delta\phi\\ \Delta\bar{\boldsymbol{h}}_o\\ \Delta\tilde{\boldsymbol{h}}_o\end{bmatrix} = -\boldsymbol{H}_J^{-1}\begin{bmatrix}\frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\phi}\\ \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\bar{\boldsymbol{h}}_o}\\ \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial\tilde{\boldsymbol{h}}_o}\end{bmatrix}. \tag{32}$$

Given (32), we next express $\Delta\eta$ as a linear function of the first-order perturbation of the orthogonal subspace $\Delta\boldsymbol{U}_o$ given in (28). For notational simplicity, we denote $\partial A(\phi, \boldsymbol{h})/\partial\phi$ as $A'(\phi, \boldsymbol{h})$ and define the first-order partial derivative $\boldsymbol{D} = (\partial J/\partial\eta)$. We have, from (32)

$$\Delta\eta = -\boldsymbol{H}_J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)^{-1}\boldsymbol{D}(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o). \tag{33}$$

The first- and second-order partial derivatives appeared in (32) are calculated in Appendix B.

With first-order approximation

$$\boldsymbol{D}(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o) \approx \boldsymbol{D}(\phi, \boldsymbol{h}, \boldsymbol{U}_o) + \Delta\boldsymbol{D}(\phi, \boldsymbol{h}, \boldsymbol{U}_o)$$

and

$$\boldsymbol{H}_J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o) \approx \boldsymbol{H}_J(\phi, \boldsymbol{h}, \boldsymbol{U}_o) + \Delta\boldsymbol{H}_J(\phi, \boldsymbol{h}, \boldsymbol{U}_o).$$

Note that $\boldsymbol{D}(\phi, \boldsymbol{h}, \boldsymbol{U}_o) = \boldsymbol{0}$, and we can further approximate (33) as

$$\Delta\eta \approx -(\boldsymbol{H}_J^{-1} - \boldsymbol{H}_J^{-1}\Delta\boldsymbol{H}_J\boldsymbol{H}_J^{-1})\Delta\boldsymbol{D} \approx -\boldsymbol{H}_J^{-1}\Delta\boldsymbol{D}. \tag{34}$$

Combining the results in (28) and (45) in Appendix B, $\Delta\boldsymbol{D}$ can be expressed in terms of noise $\boldsymbol{N}$.

$$\begin{aligned} -\Delta\boldsymbol{D}_\phi &= \boldsymbol{h}^H\boldsymbol{C}^H\boldsymbol{Z}_z'^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{N}\boldsymbol{x}^\dagger\boldsymbol{ZCh} \\ &\quad + \boldsymbol{h}^H\boldsymbol{C}^H\boldsymbol{Z}^H\boldsymbol{x}^{\dagger H}\boldsymbol{N}^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{Z}_z'\boldsymbol{Ch} \\ -\Delta\boldsymbol{D}_{\bar{\boldsymbol{h}}_o} &= \check{\boldsymbol{C}}^H\boldsymbol{Z}^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{N}\boldsymbol{x}^\dagger\boldsymbol{ZCh} \\ &\quad + \check{\boldsymbol{C}}^T\boldsymbol{Z}^T\boldsymbol{U}_o^*\boldsymbol{U}_o^T\boldsymbol{N}^*\boldsymbol{x}^{\dagger *}\boldsymbol{Z}^*\boldsymbol{C}^*\boldsymbol{h}^* \\ -\Delta\boldsymbol{D}_{\tilde{\boldsymbol{h}}_o} &= -j\check{\boldsymbol{C}}^H\boldsymbol{Z}^H\boldsymbol{U}_o\boldsymbol{U}_o^H\boldsymbol{N}\boldsymbol{x}^\dagger\boldsymbol{ZCh} \\ &\quad + j\check{\boldsymbol{C}}^T\boldsymbol{Z}^T\boldsymbol{U}_o^*\boldsymbol{U}_o^T\boldsymbol{N}^*\boldsymbol{x}^{\dagger *}\boldsymbol{Z}^*\boldsymbol{C}^*\boldsymbol{h}^*. \end{aligned} \tag{35}$$

When the noise is i.i.d. with zero mean, it is easy to verify $E[\Delta\eta_i] = 0$. The estimator is thus unbiased. To obtain the MSE's of parameter estimates, we need to calculate $E[\Delta\eta_i^2]$. Assume the noise is independent complex white Gaussian with zero mean and variance of $2\sigma^2$. The following results hold:

$$\begin{aligned} E[\boldsymbol{a}^H\boldsymbol{N}\boldsymbol{b}\boldsymbol{c}^H\boldsymbol{N}\boldsymbol{d}] &= 0 \\ E[\boldsymbol{a}^H\boldsymbol{N}^H\boldsymbol{b}\boldsymbol{c}^H\boldsymbol{N}^H\boldsymbol{d}] &= 0 \\ E[\boldsymbol{a}^H\boldsymbol{N}\boldsymbol{b}\boldsymbol{c}^H\boldsymbol{N}^H\boldsymbol{d}] &= 2\boldsymbol{a}^H\boldsymbol{d}\boldsymbol{c}^H\boldsymbol{b}\sigma^2 \\ E[\boldsymbol{a}^H\boldsymbol{N}\boldsymbol{b}\boldsymbol{c}^H\boldsymbol{N}^*\boldsymbol{d}] &= 2\mathrm{Re}[\boldsymbol{a}^H\boldsymbol{c}^*\boldsymbol{b}^T\boldsymbol{d}\sigma^2]. \end{aligned} \tag{36}$$

The proof is straightforward, and we leave it to the readers.

Before calculating the MSE of individual parameter, define $\boldsymbol{T} = \boldsymbol{H}_J^{-1}$. Further, rewrite $\Delta\boldsymbol{D}_{\eta_i}$ in (35) as $-\Delta\boldsymbol{D}_{\eta_i} = \alpha_i^H\boldsymbol{N}\beta_i + \gamma^H\boldsymbol{N}^H\delta_i$ or $-\Delta\boldsymbol{D}_{\eta_i} = \alpha_i^H\boldsymbol{N}\beta_i + \epsilon_i^H\boldsymbol{N}^*\zeta$ with

$\alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i$, and ζ_i accordingly defined. The final MSE expressions for each parameter estimate are

$$E[\Delta\eta_i^2] = 4\sigma^2 \mathrm{Re}\Bigg\{ \boldsymbol{T}(i,1)^2 \alpha_1^H \delta_1 \gamma_1^H \beta_1 + \sum_{j=2}^{(2PL-1)} \boldsymbol{T}(i,j)^2 \alpha_j^H \epsilon_j^* \beta_j^T \zeta_j + \sum_{j=2}^{(2PL-1)} (\boldsymbol{T}(i,1)\boldsymbol{T}(i,j)\alpha_1 \epsilon_j^* \beta_1^T \zeta_j + \boldsymbol{T}(i,j)\boldsymbol{T}(i,1)\alpha_j^H \delta_1 \gamma_1^H \beta_j) + \sum_{j=2}^{(2PL-2)} \sum_{k=j+1}^{(2PL-1)} 2 * \boldsymbol{T}(i,j)\boldsymbol{T}(i,k)\alpha_j^H \epsilon_k^* \beta_j^T \zeta_k \Bigg\} \quad i = 1, \cdots, n. \tag{37}$$

It is important to point out that though seemingly complicated, the above MSE expression only contains physical parameters of the system and thus allows us to predict the performance of the algorithm directly by pluging the system parameters including the true carriers and channels.

B. The CRB

The *deterministic* CRB is derived here to compared the performance of proposed algorithm with the MLE.

For convenience, we modify our data model in (7) by merging the term $e^{jkM\phi_i}$ into $\boldsymbol{w}_i$ and rewrite it as

$$\boldsymbol{y}(k) = \sum_{i=1}^{P} \boldsymbol{w}_i(k)\boldsymbol{s}_i(k) + \boldsymbol{n}(k) \tag{38}$$

where $\boldsymbol{W}(k) = [\boldsymbol{w}_1, \cdots, \boldsymbol{w}_P]\Omega^{k-1}$, $\boldsymbol{Z}_i = \mathrm{diag}[1, z_i^1, \cdots, z_i^{K-1}]$, $\Omega = \mathrm{diag}[z_1^M, \cdots, z_P^M]$, and $s(k) = [s_1(k), \cdots, s_P(k)]^T$.

Assuming deterministic unknown symbols and complex white Gaussian noise with unknown power σ^2 [26], the likelihood function of received data is given by

$$L(\boldsymbol{y}(1), \cdots, \boldsymbol{y}(N)) = \frac{1}{(\pi\sigma^2)^{MN}} \exp\left\{ \frac{-1}{\sigma^2} \sum_{k=1}^{N} [\boldsymbol{y}(k) - \boldsymbol{W}(k)\boldsymbol{s}(k)]^H \cdot [\boldsymbol{y}(k) - \boldsymbol{W}(k)\boldsymbol{s}(k)] \right\}. \tag{39}$$

The log likelihood function is thus

$$\ln(L) = \mathrm{const} - MN \ln(\sigma^2) - \frac{1}{\sigma^2} \sum_{k=1}^{N} [\boldsymbol{y}^H(k) - \boldsymbol{s}^H(k)\boldsymbol{W}^H(k)] \cdot [\boldsymbol{y}(k) - \boldsymbol{W}(k)\boldsymbol{s}(k)]. \tag{40}$$

We now construct the Fisher information matrix (FIM) by calculating the derivative of (40) with respect to $\eta = [\sigma^2 \Phi^T \bar{\boldsymbol{s}}^T(k) \tilde{\boldsymbol{s}}^T(k) \bar{\boldsymbol{h}}_o^T \tilde{\boldsymbol{h}}_o^H]$, where

$$\begin{aligned} \Phi^T &= [\phi_1, \cdots, \phi_P] \\ \bar{\boldsymbol{s}}^T(k) &= \mathrm{Re}[s_1(k), \cdots, s_P(k)] \\ \tilde{\boldsymbol{s}}^T(k) &= \mathrm{Im}[s_1(k), \cdots, s_P(k)] \\ \bar{\boldsymbol{h}}_o^T &= \mathrm{Re}[h_{1,1}, \cdots, h_{1,L-1}, h_{2,1}, \cdots, h_{P,1} \\ &\quad \cdots, h_{P,L-1}] \\ \tilde{\boldsymbol{h}}_o^T &= \mathrm{Im}[h_{1,1}, \cdots, h_{1,L-1}, h_{2,1}, \cdots, h_{P,1} \\ &\quad \cdots, h_{P,L-1}]. \end{aligned}$$

Note that in the above equation, we assume the first channel coefficient for each user to be known; therefore, they need not be estimated. Calculation of each element of the FIM is given in Appendix C. With (47) calculated in Appendix C, the FIM can be directly constructed. We can numerically compute the variance of individual parameter estimate by inverting the FIM $\mathrm{CRB}(\eta) = \mathrm{diag}(\mathrm{FIM}^{-1})$.

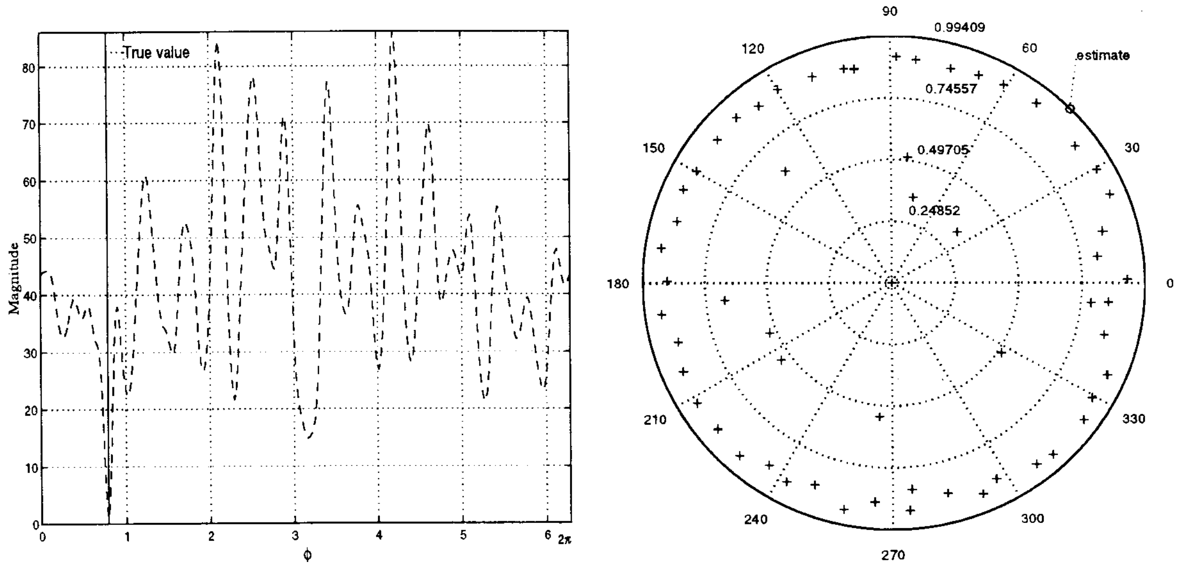
V. Numerical Results

In this section, we provide some computer simulation results to illustrate the efficacy of the proposed algorithm relative to the CRB and to verify the analytic MSE expression derived for the new algorithm. In all of the following examples, multipath channels of length 3 were used. Carrier offsets were randomly selected from $[(-\pi/32), (\pi/32)]$. The information sequence is QPSK.

First, we graphically illustrate the cost function $a(z) = \|\boldsymbol{p}(z)\|^2$ in (23) using noise-free data. Fig. 1 shows the spectrum of $a(\phi)$ and roots of $a(z)$ of a 10-user CDMA system with spreading gain $M = 32$. In Fig. 1(a), the solid line denotes the true carrier offset, while the "o" in Fig. 1(b) represents the root for the ϕ estimate. As seen, the proposed algorithm yields the exact carrier offset estimate in the absence of noise.

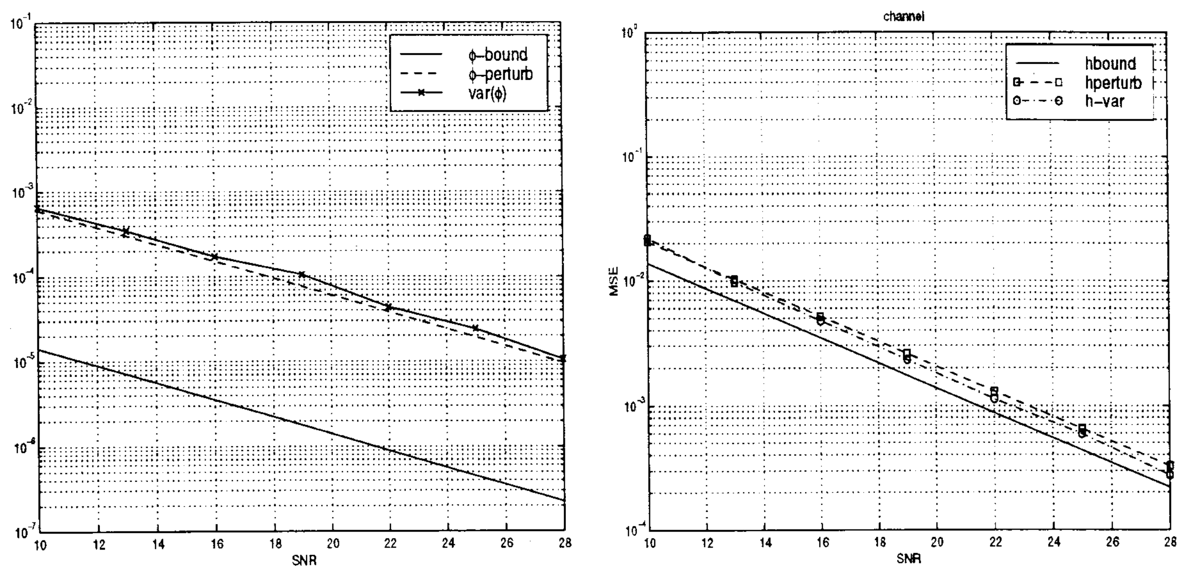
In the next example, we verify the performance predicted by perturbation analysis in Section IV under different SNR's with the following setup: $P = 2, M = 8$, $N = 20$. With SNR varying from 10–30 dB, 365 Monte Carlo trials were conducted for each SNR value. The MSE was calculated and compared to that given by theoretical prediction and the CR bound. We plotted out the MSE's for ϕ_1 and $\boldsymbol{h}_1$ in Fig. 2. The normalized MSE $\|\hat{\boldsymbol{h}}_1 - \boldsymbol{h}_1\|^2/\|\boldsymbol{h}_1\|^2$ was employed as the performance measure. As can be seen from Fig. 2, the MSE's of carrier and channel estimate obtained from simulation and perturbation analysis are very close, indicating the accuracy of the perturbation analysis for a large range of SNR values. Same results have been consistently observed in all our simulations and conducted under various setups.

Next, we show the effect of number of symbols on the performance of our method. As seen from (37), all the terms in the final MSE expression are constant except for the terms with $\boldsymbol{x}^{\dagger H}$, which is inversely proportional to the square root of signal power. Therefore, the MSE should be inversely



(a) (b)

Fig. 1. (a) Null spectrum and (b) root distribution of $a(\phi)$ and $a(z)$.



(a) (b)

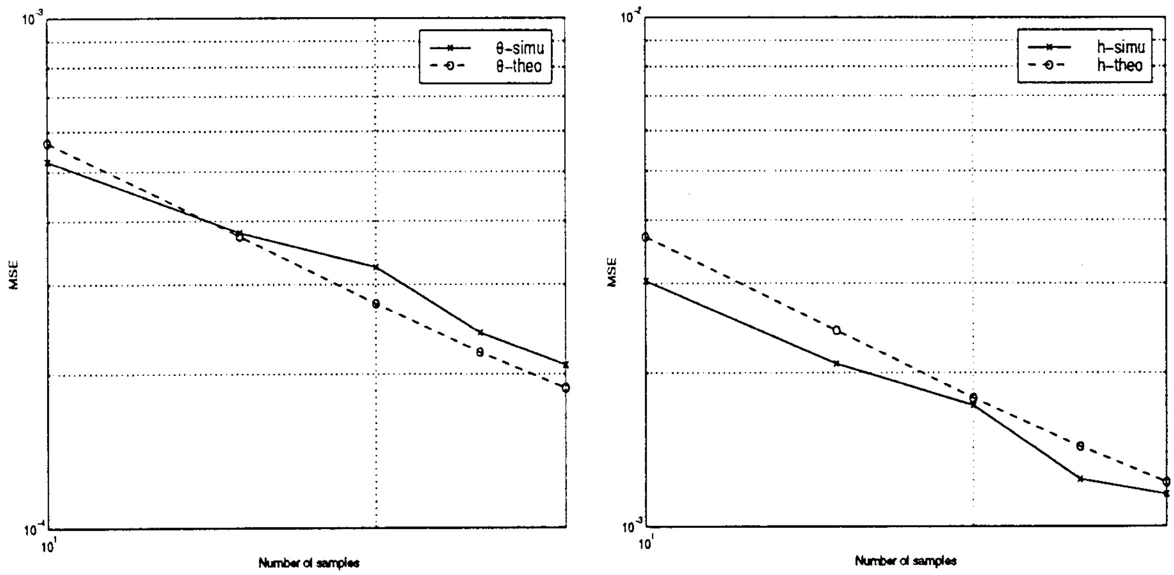
Fig. 2. MSE versus SNR. (a) Performance of ϕ estimation. (b) Performance of channel estimation.

proportional to N, which is the number of samples. Such is indeed the case in Fig. 3. The two curves in Fig. 3(b) are quite close. This also shows that the asymptotic performance of analysis using linear constraint and quadratic constrained algorithm is very similar.

Finally, with the estimated ϕ and $\boldsymbol{h}$, we constructed $\boldsymbol{W}$ in (10) and calculated the zero-forcing receiver according to (26). Signals of interest were then recovered using the zero-forcing receiver. The setup is $P = 4$, $M = 12$ with SNR $= 12$ dB. Only ten samples were used for parameter estimation. We recovered 100 symbols from the first and the second user, and each are shown in Fig. 4. The results are compared with that obtained by applying the algorithm developed in [10], where the carrier offsets were ignored. Clearly, by taking both the multipath channel and carrier offset into account, the proposed algorithm significantly outperforms the existing approach.

VI. Conclusion

A subspace based approach for joint carrier and channel estimation in CDMA systems has been presented. The proposed method, formulated in a similar way as many subspace-based methods, and, in particular, the MUSIC algorithm, exploits the intrinsic algebraic structure of CDMA signature waveforms and provides a closed-form solution for the multiuser joint estimation problem without knowledge of input signals. Performance of the proposed algorithm has been analyzed using first-order perturbation method. The CRB of the channel and carrier estimates is also derived. The performance of proposed algorithm has been demonstrated using computer simulations.



(a) (b)

Fig. 3. MSE versus number of samples. (a) Performance of ϕ estimate. (b) Performance of $\boldsymbol{h}$ estimate.

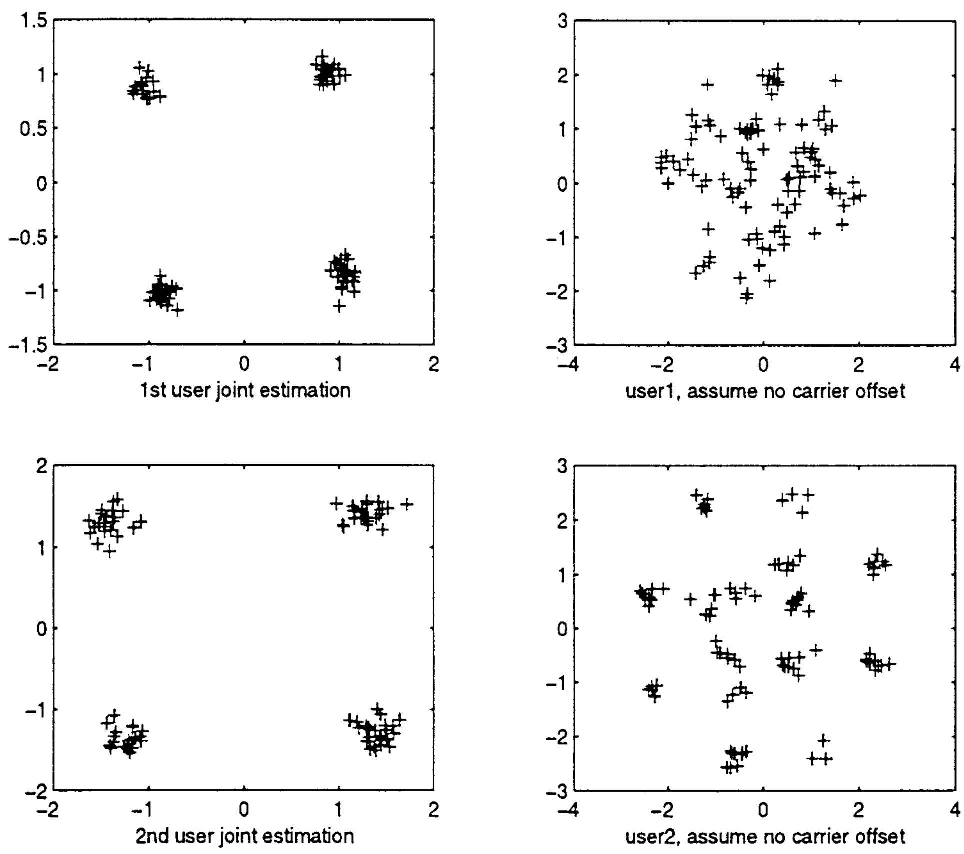


Fig. 4. Signal constellations.

Appendix A
Existence and Calculation of $\boldsymbol{G}(z)$

Before proceeding, we recall the definition of the Smith form of a polynomial matrix [27].

Definition 1: Given a $p \times r$ polynomial matrix $\boldsymbol{F}(z)$, it can always be expressed in Smith form $\boldsymbol{F}(z) = \boldsymbol{U}(z)\boldsymbol{S}(z)\boldsymbol{V}(z)$, where we have the following.

1) $\boldsymbol{U}(z), \boldsymbol{V}(z)$ are unimodular polynomial matrix in z, i.e., with constant determinant.
2) $\boldsymbol{S}(z)$ is a $p \times r$ diagonal matrix with first ρ diagonal elements $\lambda_i(z), 0 \leq i < \rho$ that are polynomials in z. The remaining diagonal elements of $\boldsymbol{S}(z)$ are zero. Here, ρ is the rank of $\boldsymbol{F}(z)$. The elements $\lambda_i(z)$ are given by $\lambda_i(z) = (\Delta_{i+1}(z)/\Delta_i(z)), \Delta_0(z) = 1$, and $\Delta_i(z)$ is the

greatest common divisor of all the $i \times i$ minors of $\boldsymbol{F}(z)$. $\lambda_i(z)$ is a factor of $\lambda_{i+1}(z)$.

Theorem 1 [28]: For a $p \times r$ polynomial matrix $\boldsymbol{F}(z)$, there exists an FIR inverse $\boldsymbol{G}(z)$ that satisfies (21) iff the diagonal elements of $\boldsymbol{F}(z)$'s Smith form are $\lambda_i(z) = z^{-n_i}, 0 \leq i < r$.

In our case, $p = L, r = L-1$. The above theorem asserts that if all of the $(L-1)\times(L-1)$ minors of $\boldsymbol{F}(z)$ are coprime with the exception of factor z^{n_i}, which holds almost surely in practice, there exists an FIR inverse $\boldsymbol{G}(z)$ that satisfies (21). We now calculate $\boldsymbol{G}(z)$ under the assumption that such an inverse exists.

Rewrite $\boldsymbol{F}(z)$ and $\boldsymbol{G}(z)$ in matrix polynomial form as

$$\boldsymbol{F}(z) = \sum_{i=0}^{K-1} \boldsymbol{F}(i) z^{-i}, \quad \boldsymbol{G}(z) = \sum_{i=0}^{B-1} \boldsymbol{G}(i) z^{-i}. \tag{41}$$

The time-domain equivalent of (21) is given by

$$\underbrace{[\boldsymbol{G}(0)\, \boldsymbol{G}(1) \cdots \boldsymbol{G}(B-1)]}_{\boldsymbol{G_G}} \cdot \underbrace{\begin{bmatrix} \boldsymbol{F}(0) & \boldsymbol{F}(1) & \cdots & \boldsymbol{F}(K) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{F}(0) & \cdots & \boldsymbol{F}(K-1) & \boldsymbol{F}(K) & \cdots & \boldsymbol{0} \\ \vdots & & \ddots & & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{F}(0) & \cdots & \cdots & \boldsymbol{F}(K) \end{bmatrix}}_{\boldsymbol{F_F}, [BL \times (K+B-1)(L-1)]} = \underbrace{[\boldsymbol{0} \cdots \boldsymbol{I} \cdots \boldsymbol{0}]}_{\boldsymbol{I}_o} \tag{42}$$

where $\boldsymbol{I}_o$ has $\boldsymbol{I}_{L-1 \times L-1}$ at its n_0th block.

By choosing appropriate B [the length of $\boldsymbol{G}(z)$] and the delay index n_0, $\boldsymbol{G_G}$ can be determined as the solution of (42).

The proposition below establishes the existence of an FIR inverse of $\boldsymbol{F}(z)$ in the time domain and can be utilized to directly calculate $\boldsymbol{G}(z)$.

Proposition 1: There exists a $\boldsymbol{G_G}$ that satisfies $\boldsymbol{G_G F_F} = \boldsymbol{I}_o$ iff the product $\boldsymbol{F_F^\dagger F_F}$ has the form

$$\begin{bmatrix} * & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ * & \boldsymbol{0} & * \end{bmatrix} \stackrel{\text{def}}{=} \boldsymbol{P}_n \tag{43}$$

where $*$ stands for block elements that we do not care, and $\boldsymbol{I}$ is the nth diagonal block $1 \leq n \leq (K+B-1)$.

Proof: The sufficient condition is straightforward because if we choose $\boldsymbol{I}_o$ with $\boldsymbol{I}$ at the nth position, $\boldsymbol{G_G} = \boldsymbol{I}_o \boldsymbol{F_F^\dagger}$ is a valid solution of (42).

For the necessary condition, suppose that $\boldsymbol{F_F^\dagger F_F} \neq \boldsymbol{P}_n$, $\forall n$; therefore, $\boldsymbol{I}_o \neq \boldsymbol{I}_o \boldsymbol{F_F^\dagger F_F}$ since $\boldsymbol{G_G}$ exists and $\boldsymbol{I}_o = \boldsymbol{G_G F_F}$. This implies that $\boldsymbol{I}_o \neq \boldsymbol{G_G F_F F_F^\dagger F_F} = \boldsymbol{G_G F_F}$, which contradicts with given condition $\boldsymbol{I}_o = \boldsymbol{G_G F_F}$. Thus, $\exists n$ so that $\boldsymbol{F_F^\dagger F_F} = \boldsymbol{P}_n$. □

Note in the above theorem, $\boldsymbol{F_F}$ need not to be of full column rank, which is actually the case in our algorithm.

To calculate $\boldsymbol{G}(z)$, we first choose B satisfying $B > (K-1)(L-1)$ so that the block Hankel matrix $\boldsymbol{F_F}$ constructed from $\boldsymbol{F}(z)$ is a tall matrix. Next, we identify the delay n_0 such that the n_0th diagonal block of the product $\boldsymbol{F_F^\dagger F_F}$ is $\boldsymbol{I}$. With n_0 so chosen, $\boldsymbol{G_G}$ can be calculated as

$$\boldsymbol{G_G} = \boldsymbol{I}_o \boldsymbol{F_F^\dagger}. \tag{44}$$

$\boldsymbol{G}(z)$ can be easily reconstructed from $\boldsymbol{G_G}$.

Appendix B
First- and Second-Order Derivatives of the $J(\phi, \boldsymbol{h}, \boldsymbol{U}_o)$

$$\begin{aligned} \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \phi} &= \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z'^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \boldsymbol{C} \boldsymbol{h} \\ &\quad + \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z}_z' \boldsymbol{C} \boldsymbol{h} \\ &\approx \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z'^H \boldsymbol{U}_o \Delta \boldsymbol{U}_o^H \boldsymbol{Z} \boldsymbol{C} \boldsymbol{h} \\ &\quad + \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \Delta \boldsymbol{U}_o \boldsymbol{U}_o^H \boldsymbol{Z}_z' \boldsymbol{C} \boldsymbol{h} \end{aligned} \tag{45a}$$

$$\begin{aligned} \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \bar{\boldsymbol{h}}_o} &= \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad + \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \\ &\approx \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \Delta \boldsymbol{U}_o \boldsymbol{U}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad + \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \Delta \boldsymbol{U}_o^* \boldsymbol{U}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \end{aligned} \tag{45b}$$

$$\begin{aligned} \frac{\partial J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \tilde{\boldsymbol{h}}_o} &= j \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad - j \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \\ &\approx j \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \Delta \boldsymbol{U}_o \boldsymbol{U}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad - j \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \Delta \boldsymbol{U}_o^* \boldsymbol{U}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \end{aligned} \tag{45c}$$

$$\begin{aligned} \frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \phi^2} &= \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z''^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \boldsymbol{C} \boldsymbol{h} \\ &\quad + 2 \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z'^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z}' \boldsymbol{C} \boldsymbol{h} \\ &\quad + \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z}_z'' \boldsymbol{C} \boldsymbol{h}, \end{aligned} \tag{45d}$$

$$\begin{aligned} \frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \phi \partial \bar{\boldsymbol{h}}_o} &= \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z'^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad + \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}'^* \check{\boldsymbol{C}}^* \quad (45\text{e}) \\ &\quad + \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z}' \check{\boldsymbol{C}} \\ &\quad + \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}'^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \end{aligned} \tag{45f}$$

$$\begin{aligned} \frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \phi \partial \tilde{\boldsymbol{h}}_o} &= j \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z'^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}} \\ &\quad - j \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}'^* \check{\boldsymbol{C}}^* \\ &\quad + j \boldsymbol{h}^H \boldsymbol{C}^H \boldsymbol{Z}_z^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z}' \check{\boldsymbol{C}} \\ &\quad - j \boldsymbol{h}^T \boldsymbol{C}^T \boldsymbol{Z}'^T \hat{\boldsymbol{U}}_o^* \hat{\boldsymbol{U}}_o^T \boldsymbol{Z}^* \check{\boldsymbol{C}}^* \end{aligned} \tag{45f}$$

$$\frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \bar{\boldsymbol{h}}_o^2} = 2\, \mathrm{Re}[\check{\boldsymbol{C}}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}}] \tag{45g}$$

$$\frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \bar{\boldsymbol{h}}_o \partial \tilde{\boldsymbol{h}}_o} = -2\, \mathrm{Im}[\check{\boldsymbol{C}}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}}] \tag{45h}$$

$$\frac{\partial^2 J(\phi, \boldsymbol{h}, \hat{\boldsymbol{U}}_o)}{\partial \tilde{\boldsymbol{h}}_o^2} = 2\, \mathrm{Re}[\check{\boldsymbol{C}}^H \boldsymbol{Z}^H \hat{\boldsymbol{U}}_o \hat{\boldsymbol{U}}_o^H \boldsymbol{Z} \check{\boldsymbol{C}}]. \tag{45i}$$

Note that the first derivatives have already been expressed in terms of $\Delta \boldsymbol{U}_o^H$ by keeping only the first-order terms and using the fact that $\boldsymbol{U}_o^H \boldsymbol{Z} \boldsymbol{C} \boldsymbol{h} = \boldsymbol{0}$.

APPENDIX C
CALCULATION OF CRAMÉR–RAO BOUND

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{MN}{\sigma^2} + \frac{1}{\sigma^4} \sum_{n=1}^{N} \boldsymbol{n}^H(k)\boldsymbol{n}(k) \tag{46a}$$

$$\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)} = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{n}(k)] \tag{46b}$$

$$\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(k)} = \frac{2}{\sigma^2} \operatorname{Im}[\boldsymbol{W}^H(k)\boldsymbol{n}(k)], \tag{46c}$$

$$\frac{\partial \ln L}{\partial \phi_i} = \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Re}\left[s_i^*(k)\frac{\partial \boldsymbol{w}_i(k)}{\partial \phi_i}\,\boldsymbol{n}(k)\right] \quad i = 1, \cdots, P \tag{46d}$$

$$\Rightarrow \frac{\partial \ln L}{\partial \boldsymbol{\Phi}} = \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Re}[\boldsymbol{S}^H(k)\boldsymbol{T}^H(k)\boldsymbol{n}(k)] \tag{46e}$$

$$\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_{oi}} = \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Re}[s_i^*(k)\check{\boldsymbol{C}}_i^H \boldsymbol{Z}_i^H(k)\boldsymbol{n}(k)] \quad i = 1, \cdots, P \tag{46f}$$

$$\Rightarrow \frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_o} = \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Re}[\boldsymbol{F}(k)\boldsymbol{n}(k)] \tag{46g}$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_{oi}} &= \frac{1}{\sigma^2}\,[-js_i^*(k)\check{\boldsymbol{C}}_i^H \boldsymbol{Z}_i^H(k)\boldsymbol{n}(k) \\ &\quad + js_i(k)\check{\boldsymbol{C}}_i^T \boldsymbol{Z}_i^T(k)\boldsymbol{n}^*(k)] \\ &= \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Im}[s_i^*(k)\check{\boldsymbol{C}}_i^H \boldsymbol{Z}_i^H(k)\boldsymbol{n}(k)] \quad i = 1, \cdots, P\end{aligned} \tag{46h}$$

$$\Rightarrow \frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_o} = \frac{2}{\sigma^2} \sum_{n=1}^{N} \operatorname{Im}[\boldsymbol{F}(k)\boldsymbol{n}(k)] \tag{46i}$$

where

$$\begin{aligned}\boldsymbol{S}(k) &= \operatorname{diag}([s_1(k), \cdots, s_P(k)]) \\ \boldsymbol{T}(k) &= \left[\frac{\partial \boldsymbol{w}_1(k)}{\partial \phi_1}, \cdots, \frac{\partial \boldsymbol{w}_P(k)}{\partial \phi_P}\right] \\ \boldsymbol{F}(k) &= \begin{bmatrix} s_1^*(k)\check{\boldsymbol{C}}_1^H \boldsymbol{Z}_1^H z_1^{(1-n)M} \\ \vdots \\ s_P^*(k)\check{\boldsymbol{C}}_P^H \boldsymbol{Z}_P^H z_P^{(1-n)M} \end{bmatrix} \\ \check{\boldsymbol{C}}_i &= \boldsymbol{C}_i(:, 2:L).\end{aligned}$$

Before calculating the CRB covariance matrix, we need the following assumption and results (see [26]):

$$E[\boldsymbol{n}(k)\boldsymbol{n}^H(m)] = \sigma^2 \boldsymbol{I}, \quad E[\boldsymbol{n}(k)\boldsymbol{n}^T(m)] = 0$$
$$E[\boldsymbol{n}^H(k)\boldsymbol{n}(k)\boldsymbol{n}^T(m)] = 0$$

and

$$E[\boldsymbol{n}^H(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{n}(m)] = \begin{cases} M^2\sigma^4, & \text{for } k \neq m \\ M(M+1)\sigma^2, & \text{for } k = m \end{cases}$$

$$\begin{aligned}E&\left[\frac{\partial \ln L}{\partial \sigma}\left(\frac{\partial \ln L}{\partial \sigma}\right)^T\right] \\ &= \frac{N^2M^2}{\sigma^2} - 2\frac{N*M}{\sigma^3} \sum_{n=1}^{N} E\boldsymbol{n}^H(k)\boldsymbol{n}(k) \\ &\quad + \frac{1}{\sigma^4} \sum_{n=1}^{N} \sum_{l=1}^{N} E\boldsymbol{n}^H(k)\boldsymbol{n}(k)\boldsymbol{n}(l)\boldsymbol{n}^H(l) \\ &= \frac{NM}{\sigma^2}.\end{aligned} \tag{47a}$$

We know $\partial \ln L/\partial\sigma^2$ is uncorrelated with any other derivatives in (46). Therefore, any other covariance terms containing $\partial \ln L/\partial\sigma^2$ are 0. The rest of the elements of the FIM are given by

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(m)}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{W}(m)]\delta_{k,m} \tag{47b}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(m)}\right)^T\right] = -\frac{2}{\sigma^2} \operatorname{Im}[\boldsymbol{W}^H(k)\boldsymbol{W}(m)]\delta_{k,m} \tag{47c}$$

$$E\left[\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(m)}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{W}(m)]\delta_{k,m} \tag{47d}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \boldsymbol{\Phi}}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{T}(k)\boldsymbol{S}(k)] \tag{47e}$$

$$E\left[\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \Phi}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Im}[\boldsymbol{W}^H(k)\boldsymbol{T}(k)\boldsymbol{S}(k)] \tag{47f}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{F}^H(k)] \tag{47g}$$

$$E\left[\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Im}[\boldsymbol{W}^H(k)\boldsymbol{F}^H(k)] \tag{47h}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}}\right)^T\right] = -\frac{2}{\sigma^2} \operatorname{Im}[\boldsymbol{W}^H(k)\boldsymbol{F}^H(k)] \tag{47i}$$

$$E\left[\frac{\partial \ln L}{\partial \tilde{\boldsymbol{s}}(k)}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}}\right)^T\right] = \frac{2}{\sigma^2} \operatorname{Re}[\boldsymbol{W}^H(k)\boldsymbol{F}^H(k)] \tag{47j}$$

$$E\left[\frac{\partial \ln L}{\partial \boldsymbol{\Phi}}\left(\frac{\partial \ln L}{\partial \Phi}\right)^T\right] = \frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Re}[\boldsymbol{S}^H\boldsymbol{T}^H(k)\boldsymbol{T}(k)\boldsymbol{S}(k)] \tag{47k}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_o}\left(\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_o}\right)^T\right] = \frac{4}{\sigma^4}\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathrm{Re}[\boldsymbol{F}(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{F}^H(m)] = \frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Re}[\boldsymbol{F}(k)\boldsymbol{F}^H(k)] \tag{47l}$$

$$E\left[\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_o}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_o}\right)^T\right] = \frac{4}{\sigma^4}\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathrm{Re}[\boldsymbol{F}(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{F}^H(m)] = \frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Re}[\boldsymbol{F}(k)\boldsymbol{F}^H(k)] \tag{47m}$$

$$E\left[\frac{\partial \ln L}{\partial \boldsymbol{\Phi}}\left(\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_o}\right)^T\right] = \frac{4}{\sigma^4}\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathrm{Re}[\boldsymbol{S}^H(k)\boldsymbol{T}^H(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{F}^H(m)] = \frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Re}[\boldsymbol{S}^H(k)\boldsymbol{T}^H(k)\boldsymbol{F}^H(k)] \tag{47n}$$

$$E\left[\frac{\partial \ln L}{\partial \boldsymbol{\Phi}}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_o}\right)^T\right] = -\frac{4}{\sigma^4}\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathrm{Im}[\boldsymbol{S}^H(k)\boldsymbol{T}^H(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{F}^H(m)] = -\frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Im}[\boldsymbol{S}^H(k)\boldsymbol{T}^H(k)\boldsymbol{F}^H(k)] \tag{47o}$$

$$E\left[\frac{\partial \ln L}{\partial \bar{\boldsymbol{h}}_o}\left(\frac{\partial \ln L}{\partial \tilde{\boldsymbol{h}}_o}\right)^T\right] = -\frac{4}{\sigma^4}\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\mathrm{Im}[\boldsymbol{F}(k)\boldsymbol{n}(k)\boldsymbol{n}^H(m)\boldsymbol{F}^H(m)] = -\frac{2}{\sigma^2}\sum_{n=1}^{N}\mathrm{Im}[\boldsymbol{F}(k)\boldsymbol{F}^H(k)]. \tag{47p}$$

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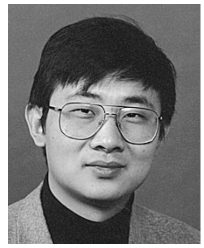

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