

# Dynamic Scheduling in Antenna Array Packet Radio \*

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## Abstract

*Efficient exploitation of the spatial diversity is fundamentally important to resource critical wireless applications. In this paper, a scheduling protocol for space-division multiple-access (SDMA) wireless packet networks [1, 2] is proposed. The new protocol is "channel-aware" in the sense that it performs scheduling based on terminals' spatial characteristics to achieve throughput multiplication and reduction of packet delays. Performance of the system is evaluated and compared to the throughput upper and lower bounds derived in the paper. Our analytic results show that with scheduling the throughput of the 2-antenna SDMA approaches its upper bound for a large terminal population.*

## 1 Introduction

One of the key resources in wireless communication is the spatial diversity provided by antenna arrays. The exploitation of the spatial diversity has been limited to physical layer using techniques such as spatial beamforming. However if the MAC protocol is designed without adaptability to the channel conditions, it has to be designed for the *worst* case scenario and as a result, there will be too much signal level margin and a significant waste of network resources.

A fundamental solution to the above problem is the "channel-aware" protocol that controls the traffic based on the spatial characteristics of the terminals.

The main contribution this paper is a scheduling protocol for slotted antenna array packet networks. We propose a scheme to increase the SDMA throughput by allocating terminals into slots of variable length based on these spatial characteristics. We show that with the proposed scheme a much more substantial improvement can be obtained than with the regular

SDMA. Another salient feature of the new protocol is that it has a maximum capacity improvement in worst case limited applications.

The remainder of this paper is organized as follows. In Section 2, we provide an overview of slotted antenna array packet networks and the conventional SDMA. Section 3 describes the proposed scheduling protocol, including a slot allocation scheme and a mechanism for slot adjustment. The performance of the protocol is analyzed in Section 4 where we derived the throughput upper and lower bounds and the asymptotic performance of the protocol. The analytic results are verified through computer simulations in Section 5. And finally in Section 6, we conclude the paper by highlighting our contributions.

## 2 Antenna Array Packet Networks

### 2.1 Spatial beamforming

In the absence of noise, the response of an  $M$ -element antenna array to a narrowband source  $s(t)$  can be written as

$$\mathbf{y}(t) \stackrel{\text{def}}{=} [y_1(t) \ y_2(t) \ \cdots \ y_M(t)]^T = \mathbf{a}\mathbf{s}(t),$$

where  $(\cdot)^T$  denotes transposition, and  $\mathbf{a} = [a(1) \ a(2) \ \cdots \ a(M)]^T$  is the array response vector (*spatial signature*).

When  $d$  terminals communicate simultaneously with the basestation, the total output of the antenna array is given by

$$\begin{aligned} \mathbf{y}(t) &= \sum_{k=1}^d \mathbf{a}_k s_k(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (1) \\ \mathbf{A} &= [\mathbf{a}_1, \dots, \mathbf{a}_d], \quad \mathbf{s}(t) = [s_1(t), \dots, s_d(t)]^T, \\ \mathbf{n}(t) &= [n_1(t), n_2(t), \dots, n_M(t)]^T \quad (2) \end{aligned}$$

where  $\mathbf{n}(t)$  is the additive noise vector and  $\mathbf{A}$  is defined as the *array manifold* whose columns are the spatial signatures. We assume (i) all signals have unit power

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and  $\|\mathbf{a}_k\| = 1$  (i.e., perfect power control), and (ii) the noise is i.i.d. with strength  $\sigma_n^2$ . Then, the covariance matrix of the array output has the form of

$$\mathbf{R}_{yy} = E\{\mathbf{y}(t)\mathbf{y}^H(t)\} = \mathbf{A}\mathbf{A}^H + \sigma_n^2\mathbf{I}. \quad (3)$$

If *spatial beamforming* [3] is used to retrieve individual signal  $\hat{s}_i(t)$ , then

$$\hat{s}_i(t) = \sum_{m=1}^M w_i^*(m)y_m(t)$$

The optimum weight vector  $\mathbf{w}_k = [w_k(1) \cdots w_k(M)]^T$  that minimize the mean squared error (MSE) of the signal estimate is given by

$$\mathbf{w}_k = \mathbf{R}_{yy}^{-1}\mathbf{a}_k,$$

in which case the output MSE and SINR respectively, of the  $k$ th signal estimate

$$\begin{aligned} MSE_k &= 1 - \mathbf{a}_k^H \mathbf{R}_{yy}^{-1} \mathbf{a}_k; \\ SINR_k &= \frac{\|\mathbf{a}_k^H \mathbf{R}_{yy}^{-1} \mathbf{a}_k\|^2}{\sum_{j \neq k}^K \|\mathbf{a}_j^H \mathbf{R}_{yy}^{-1} \mathbf{a}_k\|^2 + \|\mathbf{a}_k^H \mathbf{R}_{yy}^{-1} \mathbf{a}_k\|^2 \sigma_n^2}. \end{aligned} \quad (4)$$

The capability of simultaneous communications with multiple terminals using antenna array enables SDMA.

## 2.2 Spatial-division multiple-access

Consider the slotted packet network consists of  $K$  radio terminals actively communicating with a basestation which as  $M$  antennas.

In most practical situations  $K \gg M$ . The SDMA accommodates these terminals in both the space and the time/frequency/code domain. Built upon the slotted TDMA network, the SDMA expands the capacity by-allowing multiple (up to  $M$ ) terminals in each slot, and within each slot spatial beamforming is performed to acquire packets from multiple terminals.

Let  $\{\mathbf{a}_k\}_{k=1}^K$  be the spatial signatures of the  $K$  terminals in the system. The terminals are distributed among the  $L$  slots in SDMA and

$$\mathbf{A}_l = [\mathbf{a}_{l1} \cdots \mathbf{a}_{ld_l}]$$

denotes the array manifold of the co-slot terminals in the  $l$ th slot. The basestation performs spatial beamforming on the received co-slot signals as described in Section 2.1. According to Poor and Verdu [4], the BER of the beamformer output is given by

$$P_E = Q(SINR). \quad (5)$$

Assuming the error correction capability of a coded packet of  $N$  bits is  $t$ , the *packet success probability*

$$P_k = P(SINR_k) = \sum_{n=0}^t \binom{N}{n} P_E^n (1 - P_E)^{N-n} \quad (6)$$

where  $SINR_k$  is the beamforming output SINR of the  $k$ th terminal.

The *throughput* of each terminal in SDMA can be defined accordingly.

**Definition 1** Let each terminal in SDMA transmit packets in one out of  $L$  slots, where  $L$  is the total number of slots in a fixed frame. The throughput of the  $k$  terminal is defined as

$$T_k = \frac{1}{L} P_k, \quad k = 1 \cdots, K, \quad (7)$$

and the average throughput of the SDMA,

$$T_{ave} = \frac{1}{K} \sum_{k=1}^K T_k. \quad (8)$$

Figure 1 illustrates the performance gain of SDMA by comparing the packet throughput in a 4 antenna, 36-terminal, 12-slot SDMA system with that of a single-antenna 36-slot TDMA system (only one terminal allowed in each time slot). The TDMA packet success rate is set at 0.95. The spatial signatures are generated as independent Gaussian vector (normalized to unit power). SDMA is fundamentally advantageous when compared to pure TDMA, in that significant gains in throughput are obtained for all terminals. On the other hand, two shortfalls of conventional SDMA are evident:

1. The variation of individual terminals' performance is very large. Since the capability of most wireless systems is worst case limited, the throughput improvement over pure TDMA is actually quite limited.
2. If the number of slots is fixed, this can lead to too many or too few terminals in each slot. Either way the spatial reuse gain will be significantly reduced.

Note that *optimum* beamforming is assumed in spatial separation, hence the above problems *cannot* be cured with physical layer treatment.

### 3 Scheduling Protocol

The objective of MAC layer scheduling is to increase the efficiency of SDMA through intelligent MAC level planning. In particular, the protocol (i) systematically assigns “most-orthogonal” terminals to the same time slot so that the worst case and average performance is optimized; (ii) varies the total number of slots in a fixed frame so that that number of co-slot terminals is maintained at the level that is most suitable for spatial beamforming.

For simplicity we invoke the following assumptions:

1. All terminals spatial signatures are known to the basestation
2. All  $K$  terminals have packets to transmit. Each terminal transmits through one out of the  $L$  slot.
3. All packets have the same priority

#### 3.1 Terminal allocation

The objective of terminal allocations is to distribute the- $K$  terminals with spatial signature  $\{\mathbf{a}_k\}_{k=1}^K$  into  $L$  slots so that the throughput of the resulting SDMA system is optimum. This is accomplished as follows.

- At the beginning, the basestation assigns the first  $L$  terminals to the  $L$  open slots. The terminal allocator keeps a record of the array manifold of each slot (after the first step only one terminal in each slot)  $\mathbf{A}_l = [\mathbf{a}_{l1} \cdots \mathbf{a}_{ld_l}]$ ,  $l = 1 \cdots L$ .
- For each additional terminal with spatial signature  $\mathbf{a}_{new}$ , the basestation make a channel-aware assignment based on a pre-determined criterion (described below). For example, using the network throughput as the design metric, the new terminal is assigned to the  $j$ th slot if it has the maximum average throughput after the new terminal is added:

$$T_{ave}([\mathbf{A}_j \ \mathbf{a}_{new}]) \geq T_{ave}([\mathbf{A}_l \ \mathbf{a}_{new}]), \quad l \neq j, \quad l = 1 \cdots L.$$

- After the new terminal is assigned to the  $k$ th slot, the basestation updates the record of the assigned slot as  $\mathbf{A}_k = [\mathbf{A}_k \ \mathbf{a}_{new}]$ .
- Repeat steps 2 and 3 until all terminals are assigned.

The criterion used here is to minimize the maximum cross-correlation between co-slot terminals’ spatial signatures over all slots. Although the beamforming performance is nonlinearly related to the covariance matrix of  $\mathbf{A}$ , simulation shows that this criterion works well at low complexity.

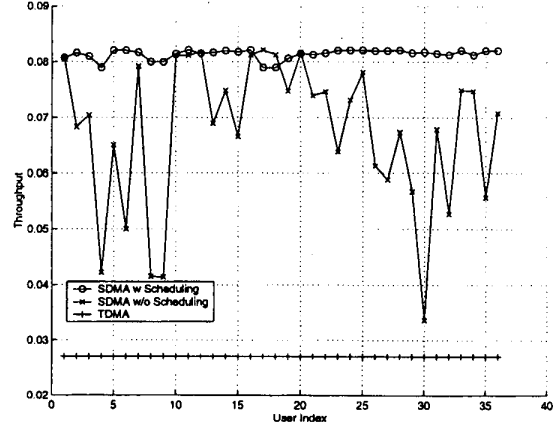


Figure 1: Throughput performance of TDMA, SDMA, and SDMA with scheduling

Figure 1 shows that with simple MAC layer planning, the system throughput increases substantially across the board. More importantly, the variance of the terminals’ throughput becomes so small. This is particularly important to wireless applications whose capacity is usually worst case limited.

#### 3.2 Slot adjustment

The efficacy of terminal allocation evidently depends on the number of slots in a frame. For optimum scheduling the protocol in principle adjust the slots size so that as many co-slot terminals can be accommodated without causing breakdown in throughput.

To find the best  $L$ , the protocol simply evaluating all possible  $L$  values and finding the the optimum frame partition.

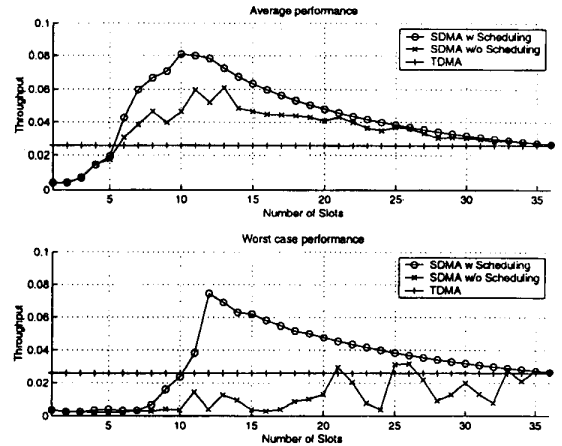


Figure 2: Throughput vs. number of slots

Figure 2 shows that the throughput for different  $L$  values given the setup in Figures 1 and 1. This verifies the importance of slot adjustment since in many cases the optimum  $L$  value may be quite different from  $K/M$ .

The “SDMA with scheduling” scheme implicitly assumes that all terminals have packets to transmit at all times. In real applications the number of terminals changes every time a terminal becomes active/inactive. Under these scenarios it may not be reasonable to perform fully scale scheduling on all terminals every time there is a change in the network traffic. One straightforward strategy is allocating only the newly active terminals based on their spatial signatures.

We shall term this protocol as “SDMA with partial scheduling”.

## 4 Analysis

In this section we analyze the efficiency of the SDMA with scheduling protocol by establishing its performance upper and lower bounds and investigating its behavior with an infinite population.

### 4.1 Upper bound and lower bound

The following lemma defines the performance upper bound of  $d$  co-slot terminals.

**Lemma 1** *Let  $M$  be the number of antenna elements,  $d$  the number of terminals sharing one slot,  $\{\mathbf{a}_i\}_{i=1}^d$ ,  $\|\mathbf{a}_i\| = 1$  the spatial signatures associated with these terminals, and  $\sigma_n^2$  the power of the additive white noise. The maximum MSE of the beamformer outputs is lower bounded by*

$$MSE_{max} \geq \begin{cases} \frac{\sigma_n^2}{1+\sigma_n^2} & d \leq M \\ \frac{d-M+M\sigma_n^2}{d+M\sigma_n^2} & d > M \end{cases} \quad (9)$$

*The equality holds when  $\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_d]$  are column orthogonal for the case of  $d \leq M$ , and row orthogonal for the case of  $d > M$ .*

With Lemma 1, we now consider the case of SDMA with variable slot length.

**Theorem 1** *For a  $K$ -terminal,  $M$ -antenna system, the worst case throughput upper bound and lower bound are given by*

$$\frac{1}{K}P(1/\sigma_n^2) \leq T_{wrst} \leq \frac{1}{L}P(1/\sigma_n^2), \quad L = \lceil \frac{K}{M} \rceil$$

## 4.2 Asymptotic optimality

When the number of users to be scheduled becomes larger, it is more likely for the scheduler to group orthogonal terminals together and yield better performance. The performance of the scheduler for a 2-antenna system is given by the following theorem:

**Theorem 2** *For a two-antenna system, the average throughput of SDMA with scheduling reaches the throughput upper bound for an infinite terminal population.*

At this moment we are unable to provide a rigorous proof for the higher dimensional case ( $M > 2$ ). However extensive simulations show the throughput upper bound is reached when the number of terminals is large regardless of  $M$ .

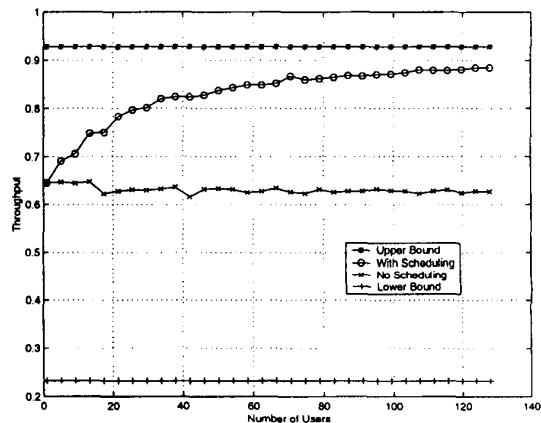


Figure 3: Asymptotic performance of SDMA with scheduling

Figure 3 demonstrates the asymptotic performance of SDMA with scheduling. With 4 antennas, the throughput upper bound per time-spatial slot is 0.92.

## 5 Performance Evaluation

Our analysis so far has premised on all terminals having packets to transmit at all time. In this Section we present some performance results for the SDMA schemes (without scheduling, with scheduling, and with partial scheduling) under bursty data traffic models. We assume

1. A fixed number of users in the system.
2. The packets arrive at each terminal in Poisson distribution with rate  $\lambda$  when this terminal is in ON period. No packet arrives in OFF period.

The length of ON and OFF period obeys certain distributions.

- The spatial signature of each terminal varies slowly in comparison with the length of ON period.

We simulate a basestation with 4 antennas serving 256 terminals. The length of OFF period is exponentially distributed with mean time 1024. The ON period has fixed length 8. During the ON period, packets arrive according to the Poisson process at rate 2.5. The average aggregate traffic rate  $\lambda$  is given by

$$\lambda = \frac{\lambda_{ON} \times \text{Number of Users} \times T_{ON}}{T_{ON} + T_{OFF}}$$

For SDMA without scheduling the number of slots in a frame is fixed at 16. The noise power is set at a level such that the packet success rate for TDMA is 0.95.

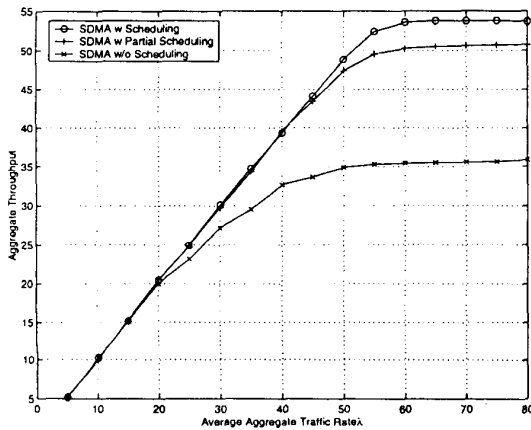


Figure 4: Throughput with burst traffic

Figure 4 and Figure 5, respectively, show the throughput and delay of the three SDMA schemes as functions of the aggregate traffic rate  $\lambda$ . For low traffic, the rise in throughput is approximately linear with the traffic rate for both schemes. As the traffic increases, the effect of scheduling becomes more and more evident. The throughput of SDMA with scheduling has a throughput gain of almost 60% over that of conventional SDMA (without scheduling). The improvement in delay characteristics due to scheduling are as significant as that in throughput. As noticed from the figure is that there is only a little performance gap between the full scheduling scheme and the partial scheduling scheme. This is particularly encouraging for practical applications that require minimum system overhead.

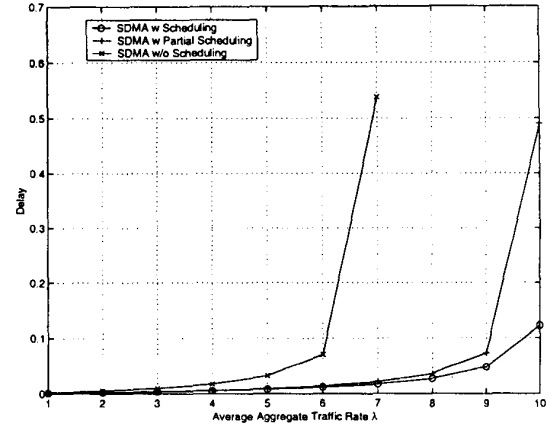


Figure 5: Delay with bursty traffic

## 6 Conclusion

In this paper we have proposed a scheduling scheme for space-division multiple access to improve the performance of a slotted antenna array packet network. The protocol judiciously selects co-channels terminals based on their spatial signatures to enable significant improvement in the system throughput-delay characteristics. We also analyzed the performance of the protocol by deriving its throughput upper and lower bounds and establishing its asymptotic optimality.

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