A Decorrelating RAKE Receiver for CDMA Communications Over Frequency-Selective Fading Channels

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***Abstract*— Recent studies show that multiuser detection in code-division multiple-access (CDMA) communications can be performed without explicit knowledge of users' channel characteristics in a frequency-selective fading environment [1], [2]. However, the computations of these blind approaches are an order of magnitude higher than existing adaptive minimum output energy (MOE) receivers [3], [4] which require at least knowledge of the desired user's channel response. Although the high-complexity problem can be alleviated by constrained adaptive filtering [5], the tradeoff is a significant drop in receiver performance, especially when the multipath pattern is time varying. In this paper, we present an adaptive receiver for CDMA communications over frequency-selective, and possibly time-varying, wireless channels. A salient feature of the new receiver is that it has complexity and performance comparable to that of the well-known MOE receivers [3], [4], and yet requires no knowledge of the desired user's channel characteristics.**

***Index Terms*— Adaptive filtering, blind detection, multiuser CDMA.**

I. Introduction

CODE-DIVISION multiple access (CDMA) is seen as one of the generic next-generation signal access strategies for wireless communications. In addition to its interference immunity and bandwidth efficiency, CDMA has shown real promise for personal communications system (PCS) applications due to its adaptability to dynamic traffic patterns in a mobile environment. If all mobile radio signals arriving at the base station are synchronized to within a fraction of a chip-time interval, synchronous CDMA (S-CDMA) systems can enhance the bandwidth efficiency to a greater degree by employing orthogonal codewords [6].

Despite its promises, S-CDMA systems have fundamental difficulties when utilized in a frequency-selective fading environment. In particular, all CDMA signals are subject to multipath induced interchip interference (ICI), leading to increased cross correlation between users' effective signature waveforms.[1] When this occurs, signal reception using matched filters suffers severe performance degradation due to mutual interference. Although a standard RAKE receiver [7], [8] can be employed to combine multipath components for a desired user, it is inherently a single-user receiver and thus is ineffective in a near–far situation [6].

Multiuser detection provides a fundamental solution to the mutual interference problem in CDMA communications [9]–[14]. While existing multiuser receivers, such as the decorrelating receiver and the minimum mean square error (MMSE) receiver, offer superior performance with linear complexity, their proper operation is premised upon explicit knowledge of the users' (or at least the desired user's) effective signature waveforms, which vary with the multipath channel characteristics. Although probing signals can be used to estimate the effective signature waveforms periodically, such methods may not be affordable due to the large numbers of cochannel users and the fast varying nature of the channels. In these scenarios, there is an evident need for blind multiuser reception which is capable of suppressing interference and recovering message symbols without the use of training sequences.

The feasibility of blind effective signature waveform estimation has been shown recently by several researchers [1], [2]. However, most blind estimation algorithms proposed to date involve computationally intensive operations such as the SVD and thus may be prohibitive in practice. To overcome this difficulty, Tsatsanis [5] developed a constrained adaptive filter based on a minimum output energy (MOE) criterion developed in [3] and [4]. Compared to the subspace methods, the algorithm significantly reduces the total complexity without requiring additional information. Its disadvantage, on the other hand, is that its performance hinges upon energy of the direct path signal and thus may not be reliable when used in time-varying channel.

Motivated by the advances in constrained adaptive CDMA receivers [3]–[5], in this paper we investigate the possibility of high-performance blind reception for CDMA communications over frequency-selective fading channels. In particular, we propose an adaptive receiver that can operate in a frequency-selective fading environment with knowledge of the desired

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[1] Here, we define the *effective signature waveform* as the convolution of the unknown channel and the known transmitting spreading waveform.

user's spreading code but not its channel characteristics. The self-adaptive receiver combines the advantages of [3]–[5] and overcomes their limitations. The new receiver is termed the decorrelating-RAKE (D-RAKE) receiver for its ability in decorrelating multiuser interference and combining desired signals stemmed from the same source [15]. Main features of the D-RAKE receiver include: 1) low complexity; 2) high performance—comparable to that of the adaptive MOE receivers [3], [4]; and 3) blindness in the sense that it does not require knowledge of the desired user's channel characteristics. Its extension to antenna array CDMA systems is straightforward.

It should be noted that some of the results presented herein parallel those of [16]. The improved MOE receiver proposed by Tsatsanis and Xu operates within the subspace defined by the desired user's spreading waveform and guarantees a constant response of the signal of interest. Similar to the algorithm in the present paper, the receiver in [16] takes advantage of signals from all paths and provides strong fading resistance in a frequency-selective environment. However, the cost function in [16] involves a matrix inversion. Thus, compared with our proposed algorithm, the receiver in [16] has a higher implementational cost.

The rest of the paper is organized as follows. Section II presents a data formulation with which the self-adaptive reception problem is studied. The proposed D-RAKE receiver is introduced in Section III, and its performance is analyzed in Section IV. The efficacy of the new receiver is demonstrated in Section V with simulation results. The paper is then concluded in Section VI.

II. Problem Formulation

As a general notational convention, matrices (in capital letters) and vectors (in lower case) will be in boldface. The symbols $E[\cdot]$, $(\cdot)^H$, $(\cdot)^T$, and $\otimes$ denote expectation, Hermitian, transpose, and convolution, respectively, while the symbol $\mathbf{I}(\mathbf{0})$ denotes the identity (zero) matrix or vector with proper dimension.

The baseband received signal for CDMA communications can be represented as

$$y(t) = \sum_{i=1}^{P} \sum_{m=-\infty}^{\infty} s_i(m) a_i(t - mT_s) + v(t) \tag{1}$$

where

- P number of active users;
- T_s symbol period;
- $v(t)$ additive white noise;
- $s_i(m)$ information-bearing symbol from the ith user;
- $a_i(t)$ its associated *effective signature waveform*.

In an ideal situation (no channel effect), the ith user's spreading waveform is merely the convolution of its assigned code word $\{c_i(l)\}_{i=1}^{L_c}$, and the pulse shaping filter $g(t)$: $a_i(t) = \sum_{l=1}^{L_c} c_i(l) g(t - nT)$. Here, T denotes the chip period and L_c is the code length. When orthogonal code words and the Nyquist pulse-shaping filter are employed, perfect signal recovery can be achieved by simply despreading the chip-rate samples of $y(t)$ with the users' code words. In the presence of multipath, however, the active users' effective signature waveforms are functions of the propagation channels and thus become unknown to the receiver. In particular, the effective signature waveform $a_i(t)$ is related to the user's spreading code and the channel impulse response as follows [1]:

$$a_i(t) = \sum_{l=1}^{L_c} c_i(l) h_i(t - lT) \tag{2}$$

where $h_i(t)$ here is the unknown *composite* channel response that characterizes the propagation effects including the timing offset, delays, and multipath reflections, etc. Within a short period, it is generally plausible to model $h_i(t)$ as a time-invariant finite impulse response (FIR) filter with support: $[0 \quad LT]$, for the proper value of L [8]. For simplicity, we consider a quasi-synchronous CDMA system where all multipath signals arrive within several chip durations. In this case, $L \ll L_c$.

Under the above conditions, it has been shown in [1] that the chip rate discrete-time equivalent of (2) is given by

$$\begin{aligned}\mathbf{y}(n) &= \sum_{i=1}^{P} \mathbf{a}_i s_i(n) + \mathbf{v}(n) \\ &= \underbrace{[\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_P]}_{\stackrel{\text{def}}{=}\mathbf{A}} \underbrace{\begin{bmatrix} s_1(n) \\ \vdots \\ s_P(n) \end{bmatrix}}_{\stackrel{\text{def}}{=}\mathbf{s}(n)} + \mathbf{v}(n)\end{aligned} \tag{3}$$

where $\mathbf{y}(n) = [y_1(n) \cdots y_{L_c - L}(n)]^T$ is comprised of the $L_c - L \approx L_c$ intersymbol interference (ISI) free samples within a symbol period, $\mathbf{v}(n)$ is the noise vector, $\mathbf{a}_i$ is the ith user's discrete-time effective signature waveform; and $\mathbf{A}$ and $\mathbf{s}(n)$ denote the effective signature waveform matrix and signal vector, respectively. From (2), it is not difficult to see that $\mathbf{a}_i$ can be expressed as the product of a code matrix $\mathbf{C}_i$ and the sampled channel $\{h_i(l) \stackrel{\text{def}}{=} h_i(t)|_{t=(l-1)T}\}$

$$\mathbf{a}_i = \begin{bmatrix} c_i(L) & \cdots & c_i(1) \\ c_i(L+1) & \cdots & c_i(2) \\ \vdots & \vdots & \vdots \\ c_i(L_c) & \cdots & c_i(Lc - L) \end{bmatrix} \begin{bmatrix} h_i(1) \\ \vdots \\ h_i(L) \end{bmatrix} \stackrel{\text{def}}{=} \mathbf{C}_i \mathbf{h}_i. \tag{4}$$

Equation (4) suggests that the ith user's effective signature waveform $\mathbf{a}_i$ is uniquely determined by the *known* code matrix $\mathbf{C}_i$ and the unknown *channel vector* $\mathbf{h}_i$. This observation simplifies the effective signature waveform estimation problem by reducing it to a channel vector estimation problem with reduced parameters.

Under the above framework, a subspace effective signature waveform estimate algorithm was developed in [1]. It is shown that by exploiting the structure of (4), an overdetermined linear equation set can be constructed so that $\{\mathbf{h}_i\}_{i=1}^{P}$, and consequently $\{\mathbf{a}_i\}_{i=1}^P$, can be uniquely estimated. Once the $\{\mathbf{a}_i\}_{i=1}^P$ are available, zero-forcing and perfect signal recovery becomes feasible in the absence of noise. In particular, the pseudoinverse of $\mathbf{A}$, $\mathbf{A}^\dagger$, can be used to decorrelate the received signal as $\mathbf{A}^\dagger\mathbf{y}(n) = \mathbf{s}(n)$. In noisy cases, the MMSE receiver for the ith user is given by $(\mathbf{A}\mathbf{A}^H + \sigma_n^2\mathbf{I})^{-1}\mathbf{a}_i$. Here, we assume that the signals are i.i.d. with unitary power and the noise is white with power σ_n^2.

III. Decorrelating RAKE Receivers

The subspace approach, although capable of determining all effective signature waveforms blindly, may be computationally prohibitive in practice. In this section, we study the feasibility of low complexity blind reception suitable for real-time implementation.

Notice that (3) can be fit into an antenna array framework by regarding $\mathbf{y}(n)$ as the array output vector and $\mathbf{a}_i$ the array response vector (or spatial signature) associated with the ith user. If the desired user's array response vector is available to the receiver, as assumed in [3] and [4], then minimum mean-square error (MMSE) reception can be performed using the well-known minimum output energy criterion in array signal processing [17]. This is exactly the underlying idea behind the MOE receiver proposed in [3] and [4]. In practice, $\{\mathbf{a}_i\}$ can be estimated using the subspace approach or training sequences.

Notice that unlike the antenna array system in which the array response vector is totally unknown to the receivers, there is abundant prior knowledge of the effective signature waveforms in CDMA. In particular, $\mathbf{a}_i = \mathbf{C}_i\mathbf{h}_i$ where $\mathbf{C}_i$ is known *a priori*. In the antenna array context, this is analogous to the situation where each direction-of-arrival (DOA) of the desired user is known within a complex scalar. Therefore, in principle one can extract the desired signal one by one from each direction without causing signal cancelation. The receiver proposed in the ensuing sections exploits this structure information and accomplishes decorrelating reception without explicit knowledge of $\{\mathbf{a}_i\}$ or $\{\mathbf{h}_i\}$.

Let $s_1(n)$ be the signal of interest. We rewrite the data vector (3) as

$$\mathbf{y}(n) = \mathbf{a}_1 s_1(n) + \mathbf{u}(n)$$

where $\mathbf{u}(n) = \sum_{i=2}^P \mathbf{a}_i s_i(n) + \mathbf{v}(n)$ denotes other users' interference plus noise. For simplicity, we shall drop the subscript and consider only

$$\begin{aligned}\mathbf{y}(n) &= \mathbf{a}s(n) + \mathbf{u}(n)\\ &= \mathbf{C}\mathbf{h}s(n) + \mathbf{u}(n)\\ &= [\mathbf{c}_1 \cdots \mathbf{c}_L]\begin{bmatrix} h(1)\\ \vdots \\ h(L)\end{bmatrix} s(n) + \mathbf{u}(n)\end{aligned} \tag{5}$$

in the remainder of this paper. The objective herein is to develop a low complexity/high performance receiver which can recover $s(n)$ from $\mathbf{y}(n)$ without channel information.

A. Constrained Blind Reception

To recover $s(n)$ from $\mathbf{y}(n)$, one should intuitively utilize all signal energy, while at the same time suppressing the interference and noise. Although this is also the underlying idea of a conventional RAKE receiver, where multipath combining coefficients can be estimated from the covariance matrix of despread signals without using training sequences [18], [19], the low performance barrier of a RAKE receiver is difficult to overcome.

From (5), it is seen that the desired signal can be decomposed into a linear combination of signals projected onto a set of delayed code vectors, $\{\mathbf{c}_l\}_{l=1}^L$. Taking advantage of this structure, we will show below that the selfadaptive two-stage receiver depicted in Fig. 1 can provide near optimum estimate of $s(n)$ directly from $\mathbf{y}(n)$ without introducing undue complexity.

The new receiver has two stages, with stage-1 comprised of a set of adaptive weight vectors $\{\mathbf{w}_l\}$ and stage-2 a coherent combiner. The idea is the following. Since only the channel coefficients $\{h(l)\}_{l=1}^L$ that combines the code vectors are unknown, we can first construct a special set of weight vectors to extract the desired signal along *individual* code vectors while suppressing all interference, and then constructively combine the extracted signals to achieve near-optimum signal estimation.

The extracting operation in stage-1 can be mathematically represented as

$$\mathbf{C}^H\mathbf{w}_l = [0 \cdots 1 \cdots 0]^T \overset{\text{def}}{=} \mathbf{1}_l, \qquad l = 1, \cdots, L$$

where $\mathbf{1}_l$ is a vector with all elements 0's except 1 at the lth position. Referring to Fig. 1, the output of the lth filter (or the lth arm) is given by

$$\begin{aligned} x_l(n) &= \mathbf{w}_l^H\mathbf{y}(n)\\ &= \mathbf{w}_l^H\mathbf{a}s(n) + \mathbf{w}_l^H\mathbf{u}(n)\\ &= \underbrace{\mathbf{w}_l^H\mathbf{C}}_{\mathbf{1}_l^H}\mathbf{h}s(n) + \mathbf{w}_l^H\mathbf{u}(n)\\ &= h(l)s(n) + \mathbf{w}_l^H\mathbf{u}(n)\\ &= h(l)s(n) + e_l(n)\end{aligned} \tag{6}$$

where $e_l(n)$ denotes the effective noise and interference after filtering. Clearly, $\mathbf{w}_l$ picks up the desired signal along $\mathbf{c}_l$ without considering the signals along $\mathbf{c}_1, \cdots, \mathbf{c}_{l-1}, \mathbf{c}_{l+1}, \cdots, \mathbf{c}_L$. The use of L weight vectors allow us to extract all signals at different delays.

With a total of L arms, the signal power after the first stage is $\sum_{l=1}^L |h(l)|^2 = ||\mathbf{h}||^2$. To obtain the best signal estimate, one should maximize the signal-to-interference-and-noise ratio (SINR) at the output of each arm defined as

$$\mathrm{SINR}_l = \frac{|h(l)|^2 E[s(n)s^*(n)]}{E[e_l(n)e_l^*(n)]} = \frac{|h(l)|^2}{\mathbf{w}_l^H\mathbf{R}_{\mathbf{uu}}\mathbf{w}_l}.$$

Note the output signal power is fixed at $|h(l)|^2$, maximizing SINR_l is equivalent to minimizing the output power

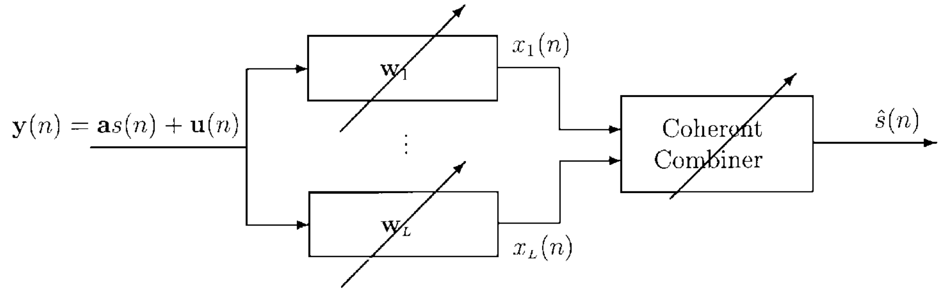

Fig. 1. The proposed D-RAKE receiver.

$E[x_l(n)x_l^*(n)]$. Therefore, the constrained MOE receiver developed in [3] and [4] is readily applied

$$\mathbf{w}_l = \arg\min_{\mathbf{w}_l} \mathbf{w}_l^H \mathbf{R}_{\mathbf{yy}} \mathbf{w}_l \qquad \text{subject to } \mathbf{C}^H \mathbf{w}_l = \mathbf{1}_l \tag{7}$$

where $\mathbf{R}_{\mathbf{yy}}$ is the autocovariance matrix of $\mathbf{y}(n)$. The adaptive rules will be outlined in the next section. It has been shown that the batch-mode MOE receiver is equivalent to an MMSE receiver under the constraint of $\mathbf{w}^H\mathbf{a} = 1$. Since $\mathbf{a}$ is unknown, each arm in the proposed receiver only provides the constrained MOE (C-MOE) estimate of the signal. Nevertheless, $\mathbf{w}_l$ is capable of eliminating all interference and perfectly recovering the signal in the absence of noise [2].

Stacking the filter outputs from all arms, $\{x_l(n)\}_{l=1}^L$, in a more compact vector form as

$$\begin{aligned}\mathbf{x}(n) &\stackrel{\text{def}}{=} \begin{bmatrix} x_1(n) \\ \vdots \\ x_L(n) \end{bmatrix} \\ &= \mathbf{h}s(n) + \begin{bmatrix} e_1(n) \\ \vdots \\ e_L(n) \end{bmatrix} \\ &\stackrel{\text{def}}{=} \mathbf{h}s(n) + \mathbf{e}(n)\end{aligned} \tag{8}$$

the second stage of the proposed receiver coherently combines the outputs from all arms to further enhance the SINR. In principle, construction of the optimum combining vector $\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{h}$ requires explicit knowledge of the channel coefficients. However, because of the interference decorrelation and noise suppression in the first stage, the total signal power in $\mathbf{x}(n)$ becomes significantly higher than that of $\mathbf{e}(n)$. In this case, $\mathbf{R}_{\mathbf{xx}} = \mathbf{h}\mathbf{h}^H + \mathbf{R}_{\mathbf{ee}} \approx \mathbf{h}\mathbf{h}^H$. This enables us to approximate the optimum combining vector using the principal eigenvector of $\mathbf{R}_{\mathbf{xx}}$, which can be obtained blindly using standard decomposition techniques [20].

The approximation here is a common tradeoff between optimality and complexity. The same technique is used in practical RAKE receivers. Note in an interference-limited CDMA environment, the performance loss suffered by the present receiver should be far less than that of the conventional RAKE receiver.

In summary, the proposed blind receiver restores the desired user's signal from the CDMA output in two steps.

Step 1: Extract the desired signal along each of the delayed code vectors using the constrained adaptive MOE receivers (7).

Step 2: Coherently combine the first stage filter outputs to further enhance the SINR of the final signal estimate.

Step 2 is critical in mobile communications. By doing so, we overcome the major difficulty of the algorithm in [5], which is essentially a single-arm receiver relying solely on $\mathbf{w}_1$. Due to channel variations, performance of the single-arm receiver is susceptible to shifts of the dominant multipath component and thus lacks the robustness for practical applications. The scheme proposed here constructively combines signals from all L arms, hence offering strong resistance against fading and timing ambiguity. The *worst case* output SINR, which usually determines the capacity of a wireless system, is greatly improved.[2] Relative to the existing MOE approaches [3], [4] which require explicit knowledge of $\mathbf{a}$, the proposed method has slightly higher complexity, depending on the maximum spread of the wireless channel. The fact that a long-delay multipath reflection is substantially weaker than a short-delay signal permits the use of a limited number of arms in the receiver without sacrificing performance.

It is worth pointing out that by replacing $\mathbf{w}_l$ with $\mathbf{c}_l$, the receiver in Fig. 1 reduces to a conventional RAKE receiver. Although resembling the RAKE in structure, the proposed method is fundamentally different from the well-known RAKE receiver which coherently combines desired signals at multiple fingers. The major difference is the fact that the proposed receiver is decorrelating in nature. After convergence to optimum values, each arm serves as a decorrelating receiver in the absence of noise. In other words

$$\mathbf{w}_{l,\text{opt}}^H[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_P] = [h_1(l) \quad 0 \quad \cdots \quad 0]$$

provided that $P + L - 1 \leq L_c - L$. Then $x_l(n) = \mathbf{w}_l^H\mathbf{y}(n) = h(l)s(n)$, which means that the signal can be perfectly restored. This is certainly not possible for a single-user RAKE receiver. In the presence of noise, each arm is a constrained MOE receiver and thus provides performance far superior to a regular RAKE receiver. To differentiate, we shall refer to the new receiver as the decorrelating-RAKE (D-RAKE) receiver in the remainder of this paper.

[2] Further results on [5] can be found in [16], where an improved algorithm is presented.

B. Adaptive Implementation

Both the weight vectors in the first stage and the coherent combining coefficients in the second stage need to be estimated from the received signals. When utilized in a nonstationary environment, it is necessary to implement the D-RAKE receiver adaptively in order to track the time-varying channels.

The constrained receiver vectors $\{\mathbf{w}_l\}_{l=1}^L$ are obtained via standard LMS algorithms. The adaptive rules are briefly reviewed below. The readers are referred to [14], [17], and [21] for more discussion.

Denote the output power of the lth arm as

$$J_{\mathrm{MOE}} = \mathbf{w}_l^H \mathbf{R}_{\mathbf{yy}} \mathbf{w}_l.$$

The objective herein is to adaptively search for a receiver vector, $\mathbf{w}_l$, that minimizes J_{MOE} subject to $\mathbf{C}^H \mathbf{w}_l = \mathbf{1}_l$.

The gradient of the cost function is given by

$$\nabla_{\mathbf{w}_l}(J_{\mathrm{MOE}}) = 2\mathbf{R}_{\mathbf{yy}}\mathbf{w}_l, \qquad l = 1, \cdots, L. \tag{9}$$

Approximating the autocorrelation matrix by the outer product of instantaneous received vector $\mathbf{y}(n)$ yields

$$\nabla_{\mathbf{w}_l}(J_{\mathrm{MOE}}) \approx 2\mathbf{y}\mathbf{y}^H\mathbf{w}_l, \qquad l = 1, \cdots, L. \tag{10}$$

In order to restrict our search direction in the constrained subspace, we need to find the projection of the gradient of the output energy onto the subspace orthogonal to $\mathbf{C}$. Upon defining the orthogonal projection matrix $\mathbf{P}_{\mathbf{C}}^{\perp} = \mathbf{I} - \mathbf{C}^H(\mathbf{C}\mathbf{C}^H)^{-1}\mathbf{C}$, we arrive at the following recursive rule:

$$\begin{aligned}\mathbf{w}_l(n+1) &= \mathbf{w}_l(n) - \mu \mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{y}(n)\mathbf{y}^H(n)\mathbf{w}_l(n)\\ &= (\mathbf{I} - \mu\mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{y}(n)\mathbf{y}^H(n))\mathbf{w}_l(n).\end{aligned} \tag{11}$$

It is easy to show if we are using $\mathbf{w}_l(0) = \mathbf{C}^{\dagger}\mathbf{1}_l$, where $\mathbf{C}^{\dagger}$ denotes the pseudoinverse of $\mathbf{C}$ as the initial weight vector, the constraint is satisfied in each iteration. The choice of step-size μ represents a compromise between rate of convergence and steady-state excess error.

Both standard and fast adaptive eigendecomposition techniques [22] can be employed to estimate the principal vector of $\mathbf{R}_{\mathbf{xx}}$ for coherent combining in the second stage. Note that since $L \ll L_c$, the computational cost [at most $O(L^3)$] at the second stage is negligible relative to that of the first stage.

IV. Performance Analysis

Several issues regarding the performance of the D-RAKE receiver are addressed in this section. While misadjustment and convergence are among the most important ones in adaptive algorithms, it is theoretically interesting to investigate the performance limit of the proposed receiver and compare it with that of the true MMSE receiver. In this section, we first evaluate the optimality of the D-RAKE receiver using the output SINR as a performance measure. The steady state behavior of the D-RAKE receiver is then studied. The results will reveal the efficacy of the new method and provide important insight into system implementation.

A. Receiver Optimality

The analysis here only provides a performance bound for the proposed algorithm in the batch-mode. The exact performance depends on the distribution of the interfering users' power, channel characteristics, and noise statistics. For simplicity, we assume that the estimation errors in the first step dominate the errors in the second step, which allows us to ignore the overall interference + noise correlation in the process of combining.

The closed-form optimum weight vector for the lth arm can be obtained via Lagrange multipliers [17]

$$\mathbf{w}_{l,\,\mathrm{opt}} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}^H\left(\mathbf{C}\mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}^H\right)^{-1}\mathbf{1}_l. \tag{12}$$

It is not hard to show that in the batch mode the weight vector in each arm will converge to its optimum value with probability 1, as the number of data observations approaches infinity. In the D-RAKE receiver, outputs from multiple arms are coherently combined. The optimum output of the D-RAKE receiver is thus given by $\sum_{l=1}^L h(l)^*\mathbf{w}_{l,\,\mathrm{opt}}^H\mathbf{y}(n)$. In effect, one can model the two-stage receiver using one receiving weight vector below:

$$\mathbf{w}_{\text{D-RAKE}} = \sum_{l=1}^{L} h(l)\mathbf{w}_{l,\,\mathrm{opt}} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}\right)^{-1}\mathbf{h}. \tag{13}$$

Correspondingly, the maximum SINR that can be achieved by the D-RAKE receiver is given by

$$\begin{aligned}\mathrm{SINR}_{\text{D-RAKE}} &= \frac{E\left[\mathbf{w}_{\text{D-RAKE}}^H\mathbf{a}s(n)s^*(n)\mathbf{a}^H\mathbf{w}_{\text{D-RAKE}}\right]}{E\left[\mathbf{w}_{\text{D-RAKE}}^H\mathbf{u}\mathbf{u}^H\mathbf{w}_{\text{D-RAKE}}\right]}\\ &= \frac{\|\mathbf{h}\|^4}{\mathbf{w}_{\text{D-RAKE}}^H\mathbf{R}_{\mathbf{uu}}\mathbf{w}_{\text{D-RAKE}}}.\end{aligned} \tag{14}$$

Similarly, it can be derived that the output SINR corresponding to the MMSE receiver, $\mathbf{w}_{\mathrm{MMSE}} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{a} = \mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}\mathbf{h}$, is given by

$$\mathrm{SINR}_{\mathrm{MMSE}} = \frac{|\mathbf{w}_{\mathrm{MMSE}}\mathbf{a}|^2}{\mathbf{w}_{\mathrm{MMSE}}^H\mathbf{R}_{\mathbf{uu}}\mathbf{w}_{\mathrm{MMSE}}} \tag{15}$$

whereas the maximum output SINR corresponding to the single-arm receiver, $\mathbf{w}_{1,\,\mathrm{opt}}$, is

$$\mathrm{SINR}_1 = \frac{|h(1)|^2}{\mathbf{w}_{1,\,\mathrm{opt}}\mathbf{R}_{\mathbf{uu}}\mathbf{w}_{1,\,\mathrm{opt}}}. \tag{16}$$

The SINR's in (14)–(16) provide the MSE upper bounds for the D-RAKE receiver, the single-arm receiver, and the adaptive MMSE receiver, respectively.

Proposition 1: When the system SNR is high

$$\mathrm{SINR}_{\mathrm{MMSE}} \geq \mathrm{SINR}_{\text{D-RAKE}} \geq \mathrm{SINR}_1.$$

In the noise-free cases, all three receivers are zero-forcing after converging to their corresponding optimum values, and thus give the same asymptotic performance. In the presence

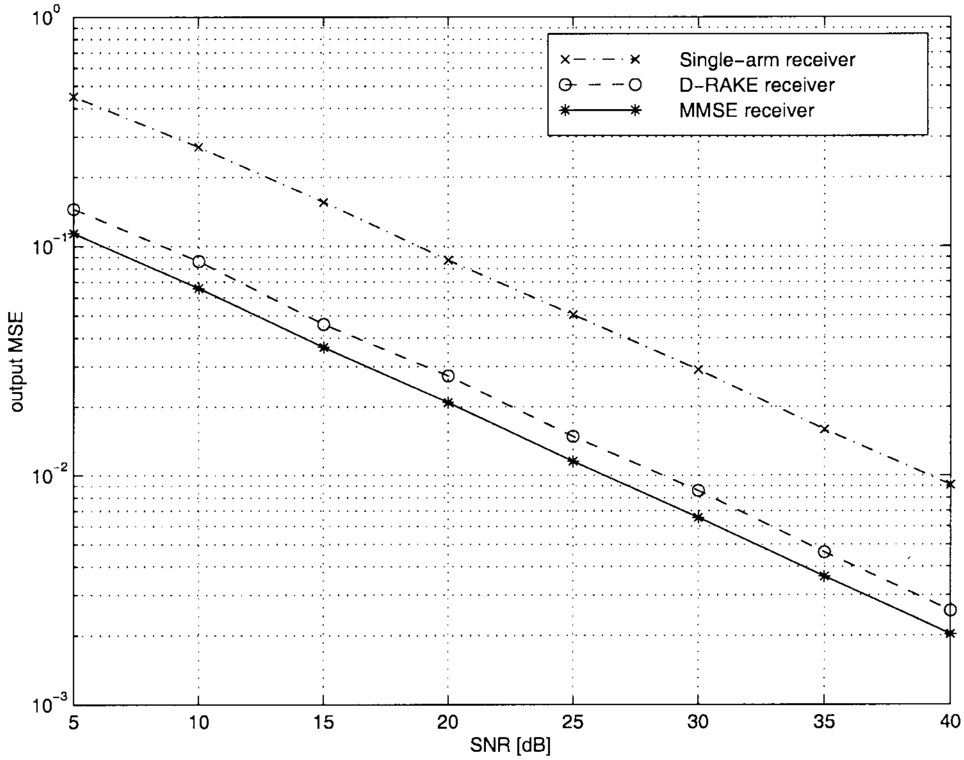

Fig. 2. SINR bound for three types of receivers.

of noise, both $\mathrm{SINR}_{\text{D-RAKE}}$ and SINR_1 are clearly upper bounded by $\mathrm{SINR}_{\mathrm{MMSE}}$. On the other hand, since the principal vector of $\mathbf{R}_{\mathbf{xx}}$ is the optimum combining vector when the system SNR is high, $\mathrm{SINR}_{\text{D-RAKE}}$ must be higher than SINR_1. The gaps between $\mathrm{SINR}_{\text{D-RAKE}}$ and $\mathrm{SINR}_{\mathrm{MMSE}}$, and SINR_1 and $\mathrm{SINR}_{\text{D-RAKE}}$, are determined by the angles between the weight vectors.

Since the single-arm receiver only uses signal output from one arm and discards the others unless there is no multipath components and the system is perfectly synchronized, i.e., $\mathbf{h} = [h(1), 0 \cdots 0]^T$, the performance of the single-arm receiver is always inferior to that of the D-RAKE receiver. The degradation in performance can be significant when the wireless system is dynamic.

For illustration, we simulated a ten-user CDMA system with frequency-selective channels and numerically calculated MSE's of the symbol estimates for the three approaches, namely, the MMSE receiver, the proposed D-RAKE receiver, and the single-arm receiver. The results are illustrated in Fig. 2 under different SNR's. As shown, in the batch-mode the D-RAKE receiver significantly outperforms the single-arm receiver. Relative to the MMSE receiver, the proposed scheme eliminates the need for training sequences without compromising system performance and simplicity.

B. Steady-State Behavior

In adaptive filtering, we are more interested in the steady-state behavior and especially the excess MSE of an adaptive algorithm. To derive the excess MSE of the proposed receiver, we first use the results in [3] to analyze the performance of the receiving filter at each arm. The effect of coherent combining on the final steady state SINR is then derived. Our analysis here follows standard steps, which is common in the study of adaptive LMS algorithms.

For the lth arm, define $\Delta\mathbf{w}_l(n) = \mathbf{w}_l(n) - \mathbf{w}_{l,\,\mathrm{opt}}$, and subtract $\mathbf{w}_{l,\,\mathrm{opt}}$ from both sides of (11), we obtain

$$\begin{aligned}\Delta\mathbf{w}_l(n+1) = &(\mathbf{I} - \mu\mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{y}(n)\mathbf{y}^H(n))\Delta\mathbf{w}_l(n)\\ &- \mu\mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{y}(n)\mathbf{y}^H(n)\mathbf{w}_{l,\,\mathrm{opt}}.\end{aligned} \tag{17}$$

Following the steps used in [3] and noticing that the same adaptation rules are used here except their projection vector is now replaced by an orthogonal projection matrix $\mathbf{P}_{\mathbf{C}}^{\perp}$, we obtain the excess MSE expression as

$$J_{l,\,\mathrm{ex}} \approx \xi_{l,\,\min} \frac{\frac{\mu}{2}\operatorname{tr}(\mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{R}_{\mathbf{yy}})}{1 - \frac{\mu}{2}\operatorname{tr}(\mathbf{P}_{\mathbf{C}}^{\perp}\mathbf{R}_{\mathbf{yy}})} \tag{18}$$

where $\xi_{l,\,\min}$ is the minimum output energy given by

$$\xi_{l,\,\min} = \mathbf{w}_{l,\,\mathrm{opt}}^H\mathbf{R}_{\mathbf{yy}}\mathbf{w}_{l,\,\mathrm{opt}} = \mathbf{1}_l^H\left(\mathbf{C}^H\mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{C}\right)^{-1}\mathbf{1}_l. \tag{19}$$

Because the output signal power is fixed at $|h(l)|^2$, the constrained MOE at the lth arm output is given by

$$J_{l,\,\min} = \xi_{l,\,\min} - |h(l)|^2. \tag{20}$$

Therefore, the steady-state MSE and the steady-state output SINR are

$$\begin{aligned}\mathrm{MSE}_{\mathrm{ss},\,l} &= J_{l,\,\min} + J_{l,\,\mathrm{ex}},\\ \mathrm{SINR}_{\mathrm{ss},\,l} &= \frac{|h(l)|^2}{J_{l,\,\min} + J_{l,\,\mathrm{ex}}},\end{aligned} \tag{21}$$

respectively.

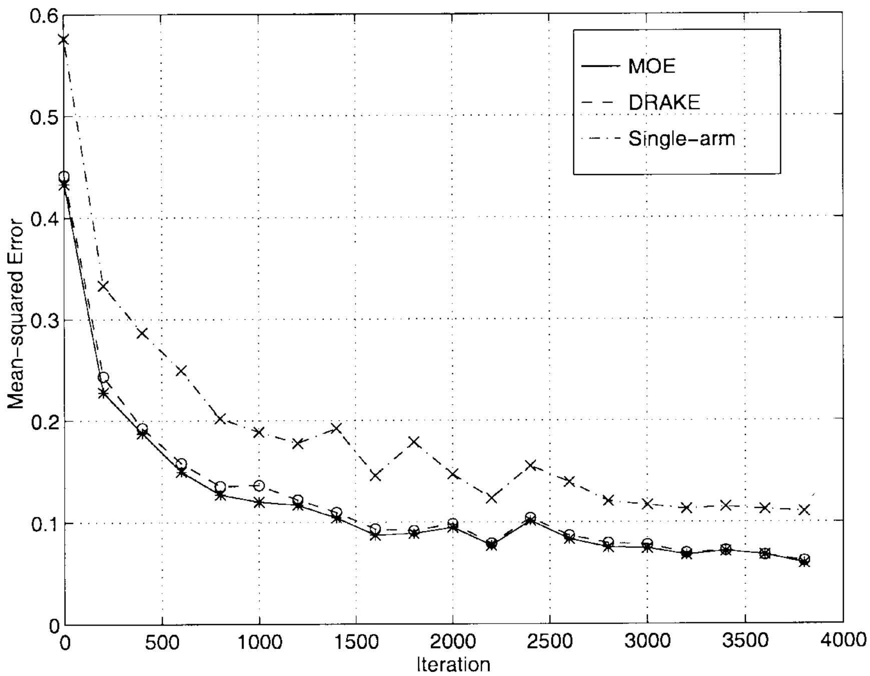

Fig. 3. Performance comparison: D-RAKE versus MOE and single-arm receivers.

To obtain the steady-state MSE of the D-RAKE receiver output, we again assume that the power of $\mathbf{e}(n)$ in $\mathbf{x}(n)$ after the first stage outputs is small relative to the signal strength. The combining vector can thus be approximated by $\mathbf{h}$. Rewrite the output from the lth arm as

$$x_l(n) = h(l)s_1(n) + e_l(n), \qquad l = 1, \cdots, L. \tag{22}$$

Here $e_l(n)$ denotes the overall error and $E[e_l(\infty)e_l^*(\infty)] = J_{l,\min} + J_{l,\mathrm{ex}}$. The output of the coherent combiner is given by

$$\begin{aligned} z(n) &= \sum_{l=1}^{L} h^2(l-1)s(n) + \sum_{l=1}^{L} h(l)e_l(n) \\ &= \|\mathbf{h}\|^2 s(n) + \sum_{l=1}^{L} h(l)e_l(n). \end{aligned} \tag{23}$$

For analytical tractability, we further assume that the excess errors at different arms are independent,[3] then the final steady state SINR is given by

$$\mathrm{SINR}_{\mathrm{ss,\,DRAKE}} = \frac{\|\mathbf{h}\|^4}{\sum_{l=1}^{L} |h(l)|^2 (J_{l,\min} + J_{l,\mathrm{ex}})} \tag{24}$$

where $J_{l,\mathrm{ex}}$ and $J_{l,\min}$ are given in (18) and (20), respectively.

Although several seemingly strong assumptions are invoked in the above derivation, the final result in (24) matched well with our simulation results under different setups. The expression of the steady-state SINR provides an important guideline in the design and implementation of the D-RAKE receiver.

[3]This is certainly not valid in general, but is frequently used in the performance analysis of adaptive filtering.

V. SIMULATIONS

Computer simulations have been conducted to examine the efficacy of the proposed reception scheme. In all of the following examples, the channel responses were generated using the well-known multiray model [21], and the pulse function was raised-cosine with a roll-off factor of 0.5. For each user, the multipath delay and the number of multipath components were uniformly distributed within $[0 \quad 3T]$ and $[1 \quad 10]$, respectively. The number of first-stage weight vectors in the proposed D-RAKE receiver was thus fixed at three. The spreading factor is set to be 32 and power control within 3 dB was assumed. The MSE, i.e., $E\{|s(n) - \hat{s}(n)|^2\}$, is used as the performance measure.

The first case involves ten almost-synchronized CDMA users. Performance of three types of receivers, namely, the proposed D-RAKE receiver, the single-arm receiver [2], and the MOE [3], [4], were compared and the results are illustrated in Fig. 3.[4] The MSE's of the D-RAKE receiver outputs are nearly identical to that of the MOE receiver, whereas the MSE's of a single-arm receiver are consistently higher. It is also observed that all three approaches converge rapidly at almost the same rate. After 3000 iterations, all three receivers reached the steady-state and the gap between the performance of the D-RAKE receiver and the MOE receiver almost vanished.

Fig. 4 compares the performance of the conventional RAKE receiver to that of the MOE and the D-RAKE receivers in a 15-user setup. Not surprisingly, both multiuser receivers offer significantly better signal estimates than the conventional RAKE receiver. Further, the performance of the D-RAKE receiver is consistently close to that of the MOE receiver. The results indicate that by exploiting the structure information of the effective

[4]For our simulation of the MOE receiver it was assumed that the receiver had perfect knowledge of the effective signature waveform $(\mathbf{a}_1)$ of the desired user.

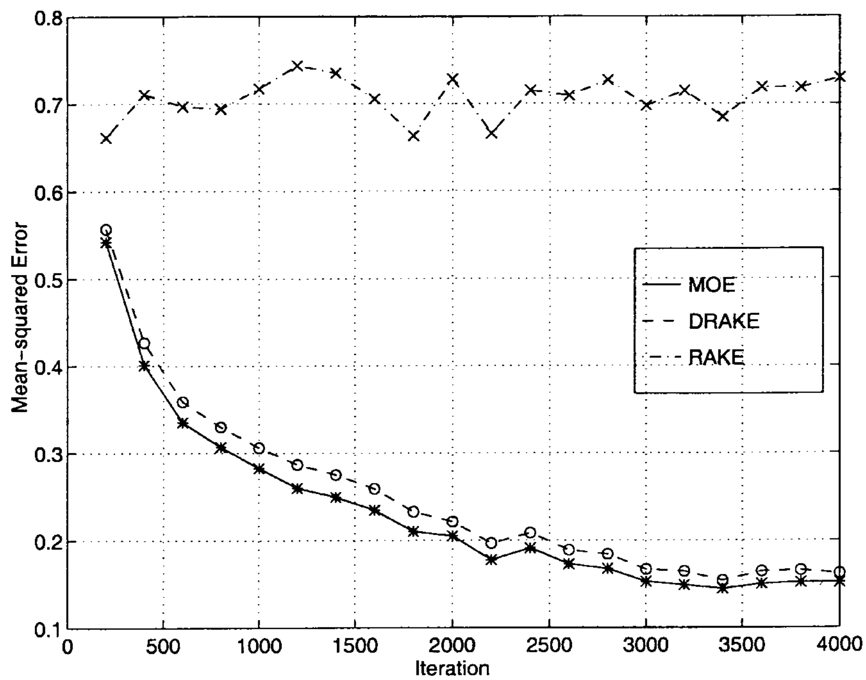

Fig. 4. Performance comparison: D-RAKE versus MOE and conventional RAKE receivers.

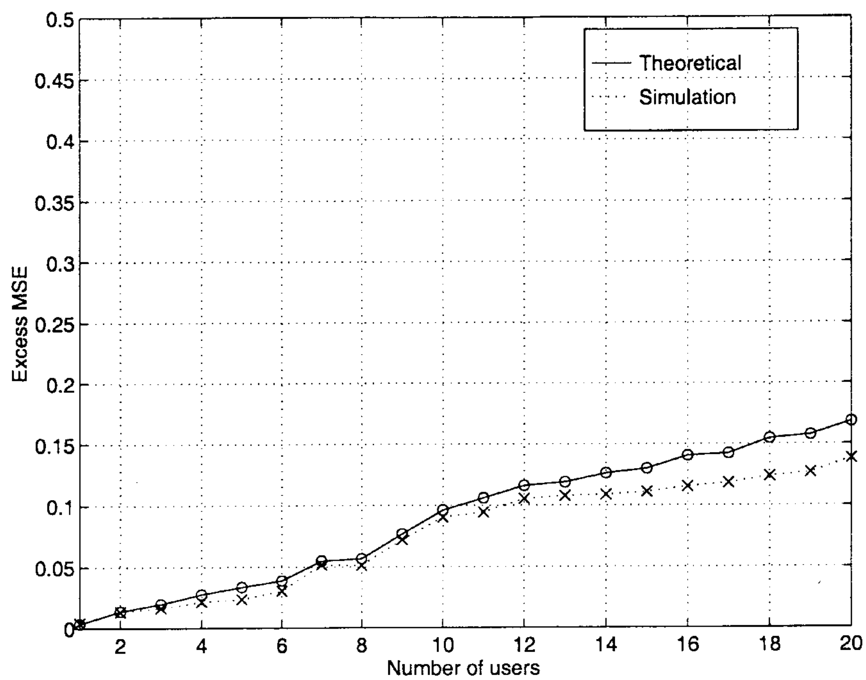

Fig. 5. Excess MSE's.

signature waveform, it is possible to approach the performance of the MOE receiver where $\mathbf{a}_1$ is known explicitly.

Numerical studies were also conducted to verify the theoretical steady-state performance predicted in Section IV. Under 3-dB SNR, we observed the excess MSE of a single-arm output for a particular user and gradually increased the number of total users in the system. As shown in Fig. 5, where the solid line and "o"s represent the theoretical predictions in Section IV-C, whereas the dotted line and "x"s represent the simulation results, the two curves match extremely well when the total number of users is under ten. As the number of users becomes large, the theoretical values tend to overpredict the excess MSE.

Also examined were the steady-state MSE's after coherent combining. We plot the simulation results versus theoretical values in Fig. 6 under the same setup as Fig. 5. The two curves matched quite well, even when the number of users was large. It implies that the deviation in the theoretical prediction for a single-arm filter was overcome by the averaging effect of coherent combining.

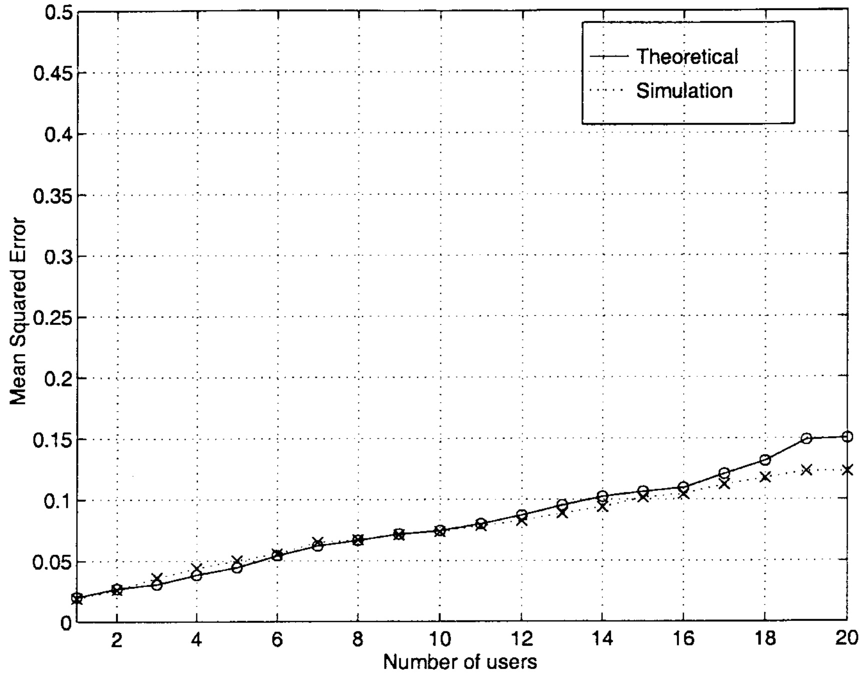

Fig. 6. Steady-state MSE's after coherent combing.

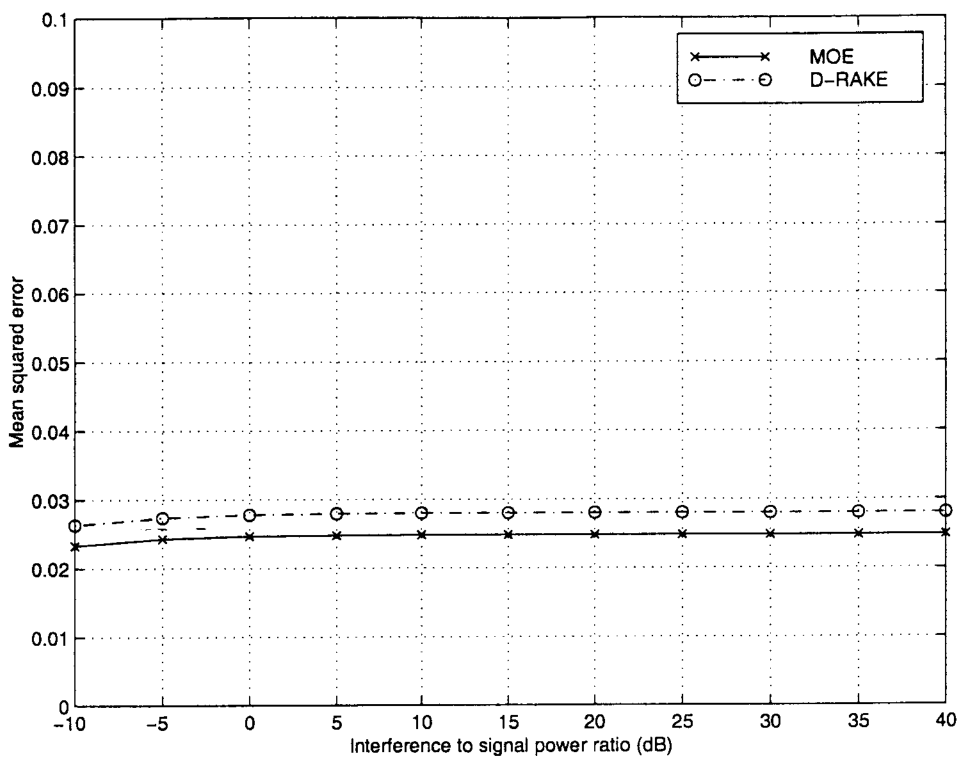

Fig. 7. Near–far resistance: D-RAKE versus MOE.

In the last example, we compare the near–far resistance of the proposed D-RAKE receiver with the near–far resistant MOE receiver. A ten-user CDMA system was simulated. We fixed the signal strength for the desired user and adjusted the power level of all the other interfering users. The SNR is set to be 3 dB. Fig. 7 plots the theoretical output MSE's of the two receivers for a range of interference to signal power ratios. It can be seen that the proposed D-RAKE receiver and the MOE receiver possess similar near–far resistance properties.

VI. Conclusion

A decorrelating RAKE receiver for CDMA communications over frequency-selective fading channels has been developed in this paper. The new blind receiver possesses important features such as low complexity and high performance, which makes it particularly useful in mobile communications. The asymptotic optimality and the steady-state behaviors of the proposed algorithm have been investigated. Our results show that the proposed receiver significantly outperforms the existing single-arm adaptive approach [5] and offers performance comparable to the MOE approaches without significant increase in complexity.

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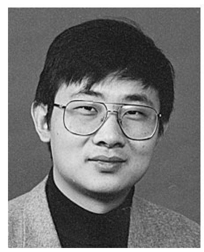

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