Blind Equalization in Antenna Array CDMA Systems

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***Abstract*—Multipath induced interchip-interference (ICI) alters waveforms of transmitted signals and presents a major obstacle to direct-sequence (DS) code-division-multiple-access (CDMA) communications. For systems with *aperiodic* pseudorandom (PN) spreading sequences, the primary way to counter fading is through employing RAKE receivers that enhance the signal-to-interference ratio (SIR) by combining multipath signals from the desired user. In this paper, we formulate a discrete-time model for antenna array CDMA systems and study the 2-D RAKE receiver problem by casting it into an optimum vector FIR equalizer design and estimation framework. A novel aspect of the present work is the full exploitation of the potential of 2-D RAKE receivers without requiring any detailed knowledge of the multipath channels.**

I. INTRODUCTION

IN addition to multiuser interference (MUI), CDMA signals suffer from multipath induced interchip interference (ICI) when wireless channels are frequency selective. For reliable CDMA communications, channel equalization is indispensable [1]–[3]. The desire for high-performance CDMA communications has led to the recent development of multiuser detection/equalization techniques that simultaneously exploit code and channel diversities characterized by users' signature waveforms. Many promising algorithms on, e.g., multiuser detection [4]–[6], blind signature waveform estimation [7], [8], blind adaptive detection [9], [10] and optimum beamforming [11] have been developed.

Despite their significance, the aforementioned techniques are applicable only to CDMA systems with fixed (or slowly varying) signature waveforms. Since the capacity of CDMA is interference limited, many practical systems utilize *aperiodic* PN codes to uniformly distribute the signal spectrum over the bandwidth. In the IS-95 standard, for example, PN sequences with a period of 2^{15} chips are used. Even in synchronous CDMA systems, masking sequences are often applied to separate different cell sites [12]. The aperiodicity of PN sequences, though beneficial to the capacity of CDMA communications, creates a major technological hurdle for signature waveform based reception schemes.

RAKE receivers [13], [14] that combine multipath signals have been demonstrated to be effective in alleviating multipath fading for CDMA systems with aperiodic spreading sequences.

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When multiple antennas are available at the base station, 2-D RAKE receivers can be employed to profit from the additional spatial diversity offered by the antenna array [15]. In a multireceiver setup, the multipath pattern between the ith user and the M-element antenna array can generally be described by a *composite vector* FIR channel [14]:

$$\mathbf{h}_i(t) \stackrel{\text{def}}{=} \begin{bmatrix} h_{1,i}(t) \\ \vdots \\ h_{M,i}(t) \end{bmatrix} = \sum_{l=1}^{L_i} \begin{bmatrix} a_{1,i}(l) \\ \vdots \\ a_{M,i}(l) \end{bmatrix} p[t-\tau_i(l)] \stackrel{\text{def}}{=} \sum_{l=1}^{L_i} \mathbf{a}_i(l) p[t-\tau_i(l)] \quad (1)$$

where

- $p(t)$ pulse shaping function,
- $\tau_i(l)$ delay corresponding to the lth multipath signal,
- $\mathbf{a}_i(l)$ array response vector corresponding to the lth multipath signal,
- L_i total number of paths associated with the ith user.

Since the conventional coherent combining approach has inherent limitations in a near-far situation, there is evident need for more effective reception techniques for CDMA communications. Toward this end, a novel 2-D RAKE reception scheme that involves array response vector estimation and optimum beamforming has been proposed in [15]. Assuming knowledge of all multipath delays, the new approach offers considerable increase in performance over the conventional matched filter-based 2-D receivers. The problem, however, is that accurate propagation delay estimates in CDMA applications is difficult to accomplish in the presence of a large number of active users [16]. Even if the multipath timing is available at the base station, beamforming on a particular multipath signal may be prohibitive due to the discrete nature of data samples. In addition to these implementational limitations, individual multipath-based approaches may not be the most efficient way to exploit the potential of 2-D RAKE receivers. More effective methods, e.g., the frequency-domain approach by Zoltowski *et al.* [17], can be developed to simultaneously incorporate the spatial and temporal diversities.

Recent research in multiinput multioutput (MIMO) systems has led to significant advances in the use of FIR receivers for CDMA systems with stationary signature waveforms [11], [18]. Relatively limited results have been reported for systems with aperiodic PN sequences, despite its practical significance. One of the reasons may be attributed to the complexity associated with multipath parameter estimation in the presence of rich interference. Another important factor is the lack of a generally applied framework under which systemic studies can be conducted.

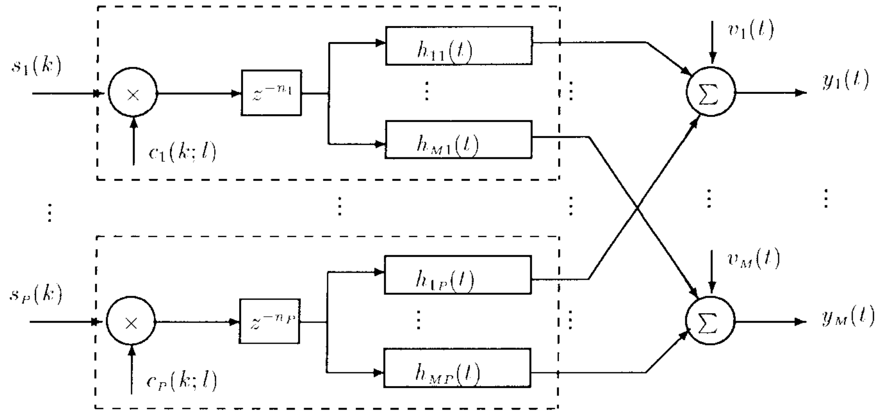

Fig. 1. Antenna array CDMA system with aperiodic spreading sequences.

The main contribution of this paper is the treatment of the full problem. In Section II of this paper, a discrete-time model for the CDMA system under consideration is presented. The resulting framework allows us to evaluate the potential of 2-D RAKE receivers for a general class of multipath channels. The optimum performance of linear FIR receivers is derived in Section III, along with the vector equalizers that can minimize the mean-square error (MSE) of signal estimates. In Section IV, we introduce a principal component method to estimate the minimum mean-square error (MMSE) equalizers directly from the antenna outputs. The new approach can significantly outperform the traditional 2-D RAKE receivers without imposing any constraint on the multipath structure. The data efficiency of the estimation is further improved in Section V, where a deterministic least-squares (LS) algorithm is developed. If the CDMA system is underloaded, i.e., the number of users is less than the number of effective channels, the deterministic approach can blindly identify zero-forcing equalizers to perfectly separate multiuser signals using a finite number of observations. Simulation results are provided in Section VI, and finally, the paper is concluded in Section VII.

II. Data Formulation

We consider an asynchronous CDMA system with P users and $M\,(M > 1)$ receivers and model it as a MIMO system (see Fig. 1). In this framework, the baseband data model for the mth receiver output is given by

$$y_m(t) = \sum_{i=1}^{P} \sum_{n=-\infty}^{\infty} d_i(n) h_{mi}(t - nT) + v_m(t) \qquad (2)$$

where the subscript i denotes the user index, T is the chip period, the chip sequences $\{d_i(n)\}_{i=1}^{P}$ are assumed to be independent of the additive noise $\{v_m(t)\}$, and the composite channel response $h_{mi}(t)$ characterizes the transfer function between the ith user and the mth antenna.

For presentation simplicity, we express the antenna outputs in a vector form as follows:

$$\mathbf{y}(t) = \sum_{i=1}^{P} \sum_{n=-\infty}^{\infty} d_i(n) \mathbf{h}_i(t - nT) + \mathbf{v}(t) \qquad (3)$$

where $\mathbf{h}_i(t)$ is defined in (1), and $\mathbf{v}(t) = [v_1(t) \cdots v_M(t)]^T$ represents the noise vector.

In direct-sequence CDMA communications, each symbol is spread into L chips. Denoting $\mathbf{c}_i(k) = [c_i(k;0) \quad c_i(k,1) \quad \cdots \quad c_i(k; L-1)]^T$ as the *aperiodic* spreading vector for the kth symbol from the ith user, then from Fig. 1

$$d_i(n) = s_i(k) c_i(k; n - kL - n_i), \qquad k = \left\lfloor \frac{n - n_i}{L} \right\rfloor \qquad (4)$$

where $n_i\,(0 \le n_i < L)$ in the above equation, which is referred to as the chip delay index, is generally available to the receivers in an asynchronous system. The exact timing (within a fraction of a chip duration) of the direct path signal is assumed to be unknown.

We invoke the following assumptions regarding the signal and additive noise that are plausible in most CDMA systems:

A1) The zero-mean noise vector $\mathbf{v}(t)$ is temporally and spatially white with

$$E\{\mathbf{v}(t)\mathbf{v}^T(t)\} = \mathbf{0}, \quad E\{\mathbf{v}(t)\mathbf{v}^H(t)\} = \sigma_n^2 \mathbf{I}$$

where $(\cdot)^H$ denotes conjugate transposition.

A2) The information symbols $\{s_i\}_{i=1}^{P}$ are i.i.d. with $E\{s_i(n)s_i^*(n)\} = 1$.

A3) The spreading codes $\{c_i(k;l)\}$ are binary i.i.d.

A4) All channels $\{\mathbf{h}_i(t)\}_{i=1}^{P}$ are linear time-invariant (LTI) with a finite duration within $[0 \quad L_c T]$.

Sampling $\mathbf{y}(t)$ at $t = nT + T/2, n = 0, 1, \cdots$ yields a discrete-time model

$$\mathbf{y}(n) \overset{\text{def}}{=} \mathbf{y}(t)|_{t=nT+T/2} = \sum_{i=1}^{P} \sum_{l=0}^{L_c-1} \mathbf{h}_i(l) d_i(n-l) \overset{\text{def}}{=} \sum_{i=1}^{P} \mathbf{h}_i(n) \otimes d_i(n) + \mathbf{v}(n) \qquad (5)$$

where $\otimes$ denotes convolution, and $\mathbf{h}_i(l) = \mathbf{h}_i(t)|_{t=lT+T/2}$, $l = 0, \cdots, L_c - 1$.

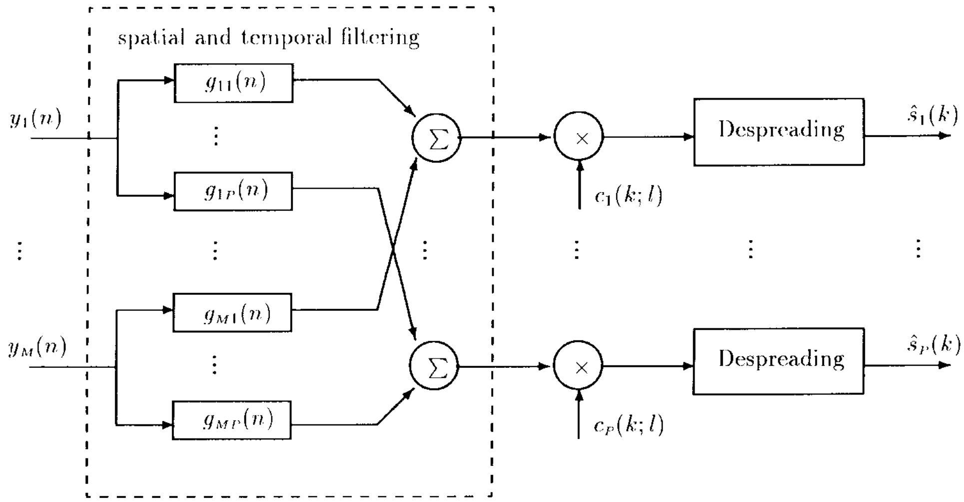

Fig. 2. 2-D RAKE receivers.

The general problem addressed in this paper is the design and estimation of linear FIR receivers to recover the transmitted signals from the discrete outputs of multiple channels. It is worth pointing out that although the above expression is formulated in the context of antenna array systems, spatial oversampling is not the only way to create multiple outputs. By stacking M consecutive samples within a chip in a vector form (temporal oversampling), single antenna outputs can be cast into the exactly same framework in (5). The multiple output structure can also be made available using a combination of both spatial and temporal oversampling. In this case, the effective number of channels M = (*number of antennas*) $\times$ (*temporal oversampling rate*) [19].

III. Discrete-Time 2-D RAKE Receivers

The RAKE receiver originally proposed by Price and Green [13] is a standard *scalar* filter that combines spread signals. After estimating the multipath structure of the channel using matched filters, the received signal is passed through a RAKE correlator that is matched to the channel response so that multipath components can be constructively combined. This idea has been extended to the antenna array case to exploit the time and space structure of the multipath environment [15].

In this paper, we consider the discrete-time 2-D RAKE receivers depicted in Fig. 2 and investigate the receiver design problem without the use of training sequences. By regarding oversampled signals as multichannel outputs sampled at the chip rate, the receiver structure under consideration is suitable for all practical systems with a sampling rate equals multiple of the chip rate.

As seen from Fig. 2, a vector FIR filter/equalizer $\mathbf{g}_i(n) = [g_{1,1}(n) \cdots g_{M,1}(n)]^T$ of order K is designated for each user to capture the diversities offered by the channels. The filter output is defined as

$$\tilde{d}_i(n-K+1) \overset{\text{def}}{=} \mathbf{g}_i(n) \otimes \mathbf{y}(n) = \sum_{k=0}^{K-1} \mathbf{g}_i^H(k)\mathbf{y}(n-k) \tag{6}$$

where $K-1$ in $\tilde{d}_i(n-K+1)$ is introduced to accommodate the maximum multipath delay. After filtering, regenerated PN sequences can then be applied for despreading. Upon defining

$$\mathbf{g}_i = [\mathbf{g}_i^H(K-1) \quad \mathbf{g}_i^H(K-2) \ \cdots \ \mathbf{g}_i^H(0)]^H, \tag{7}$$

$$\mathbf{y}_K(n) = [\mathbf{y}^H(n-K+1) \quad \mathbf{y}^H(n-K+2) \ \cdots \ \mathbf{y}^H(n)]^H \tag{8}$$

the whole reception process can be described as

$$\hat{s}_i(k) = \sum_{l=0}^{L-1} \tilde{d}_i(kL+n_i+l)c_i(k,l) = \sum_{l=0}^{L-1} [\mathbf{g}_i^H \mathbf{y}_K(kL+n_i+K+l-1)]c_i(k,l). \tag{9}$$

The above scheme can be implemented alternatively by performing despreading directly on the antenna outputs. More specifically, one may first despread $\mathbf{y}(n)$ at different delays starting at $n = n_i, n_i+1, \cdots, n_i+K-1$:

$$\mathbf{x}_{i,K}(k) \overset{\text{def}}{=} \begin{bmatrix} \sum_{l=0}^{L-1} \mathbf{y}(kL+n_i+l)c_i(k,l) \\ \vdots \\ \sum_{l=0}^{L-1} \mathbf{y}(kL+n_i+K-1+l)c_i(k,l) \end{bmatrix} = \mathbf{Y}_i(k)\mathbf{c}_i(k) \tag{10}$$

where

$$\mathbf{Y}_i(k) = [\mathbf{y}_K(kL+n_i+K-1) \ \cdots \ \mathbf{y}_K(kL+n_i+K+L-2)] \tag{11}$$

and then apply $\{\mathbf{g}_i(k)\}_{k=0}^{K-1}$ to $\mathbf{x}_{i,K}$,

$$\hat{s}_i(k) = \mathbf{g}_i^H \mathbf{x}_{i,K}(k) = \sum_{l=0}^{L-1} \tilde{d}_i(kL+n_i+l)c_i(k,l).$$

Comparing the above result with (9), these two schemes have no essential different from a signal reception viewpoint. The coefficients $\{\mathbf{g}_i(k)\}_{i=1}^{P}$ determine the efficacy of the 2-D RAKE receivers.

A. Coherent Combiners

Traditionally, matched filters are used in RAKE receivers to determine the channel responses, and signals from the desired user are coherently combined based on the channel estimates. This technique can be extended to 2-D cases by applying matched filters to all antennas. For obvious reasons, the length of the receiver K is usually set to be equal to the channel length L_c, and the receiver coefficients are simply chosen as

$$\mathbf{g}_i(l) = \mathbf{h}_i(l), \qquad l = 0, \cdots, L_c - 1.$$

A salient feature of the discrete-time coherent combiner is its simplicity, which makes it implementable in practical systems. However, a coherent combiner is essentially a single-user receiver and, therefore, has the well-known disadvantage in multiuser scenarios. In addition, it may also suffer from severe performance degradation if the delays of multipath signals are not far apart.

B. Optimum 2-D RAKE Receivers

The discrete framework in (6) renders a well-posed linear FIR filter design problem, thus allowing us to benefit from the extensive work that has been done in the context of vector channel equalization [20]–[24]. The ease with which analysis and new algorithms can be developed within this framework will manifest itself as we proceed.

In the following, we adopt the mean-square error (MSE) as the performance measure and define $\{\mathbf{g}_i(l), \quad l = 0, \cdots, K-1\}$ that minimizes

$$\begin{aligned} &E\{|\mathbf{g}_i(n) \otimes \mathbf{y}(n) - d_i(n-K+1)|^2\} \\ &\quad = E\{|\mathbf{g}_i^H \mathbf{y}_K(n) - d_i(n-K+1)|^2\} \end{aligned} \tag{12}$$

as the *MMSE equalizer* of order K. With straightforward manipulation, it is not difficult to derive that $\mathbf{g}_{i,\,mmse} = \mathbf{R}^{-1}_{\mathbf{y}_K, \mathbf{y}_K} \mathbf{r}_{i,\,K}$, where $\mathbf{R}_{\mathbf{y}_K, \mathbf{y}_K} = E\{\mathbf{y}_K(n)\mathbf{y}_K^H(n)\}$ and $\mathbf{r}_{i,\,K} = E\{\mathbf{y}_K(n) d_i(n-K+1)\}$. To obtain the explicit expression, rewrite $\mathbf{y}_K(n)$ as

$$\mathbf{y}_K(n) = \sum_{i=1}^{P} \mathbf{H}_i \mathbf{d}_{i,\,K}(n) + \mathbf{v}_K(n) \tag{13}$$

where we have (14), shown at the bottom of the page, and

$$\begin{aligned} \mathbf{d}_{i,\,K}(n) &\overset{\text{def}}{=} [d_i(n-L_c-K+1) \ \cdots \ d_i(n-K+1) \\ &\qquad \cdots \ d_i(n)]^T, \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{v}_K(n) &\overset{\text{def}}{=} [\mathbf{v}_i^H(n-K+1) \quad \mathbf{v}_i^H(n-K+2) \\ &\qquad \cdots \ \mathbf{v}_i^H(n)]^H. \end{aligned} \tag{16}$$

Then, under Assumptions A1)–A3), we obtain the following result, which optimizes the MSE under the imposed receiver structure in Fig. 2.

Proposition 1: The vector equalizer that minimizes the MSE in (12) is given by

$$\mathbf{g}_{i,\,mmse} = \mathbf{R}^{-1}_{\mathbf{y}_K, \mathbf{y}_K} \mathbf{r}_{i,\,K} \tag{17}$$

where

$$\begin{aligned} \mathbf{R}_{\mathbf{y}_K, \mathbf{y}_K} &= \sum_{i=1}^{P} \mathbf{H}_i \mathbf{H}_i^H + \sigma_n^2 \mathbf{I}, \\ \mathbf{r}_{i,\,K} &= \mathbf{H}_i \mathbf{e}_L \\ &= [\mathbf{h}_i^H(0) \ \cdots \ \mathbf{h}_i^H(L_c-1), \mathbf{0} \ \cdots \ \mathbf{0}]^H \end{aligned}$$

and $\mathbf{e}_l \overset{\text{def}}{=} [0 \cdots 0 \quad 1 \quad 0 \cdots 0]^T$ with 1 at the lth position.

The vector equalizer defined above offers the optimum performance among the class of linear FIR receivers. Unlike other alternatives, which apply separate signal processing in time and spatial domain, the MMSE equalizer takes advantage of all possible diversities in a joint fashion. Such simultaneous exploitation has been demonstrated to be instrumental to ICI and MUI suppression in a multichannel setup [19], [22], [23].

C. Performance Comparison

Under the current framework, we are able to investigate the capability of 2-D RAKE receivers without imposing any specific assumptions on the channel structure. For the purpose of comparison, we derive in the following the MSE expressions for the coherent combiner and the MMSE equalizer. The results will provide insight to the inability of existing approaches in a frequency-selective fading environment.

To begin with, decompose $\mathbf{y}_K(n)$ as follows

$$\begin{aligned} \mathbf{y}_K(n) &= d_i(n-K+1)\mathbf{r}_{i,\,K} + \\ &\underbrace{\sum_{l \neq K-1}^{K+L_c-1} \mathbf{H}_i \mathbf{e}_l \mathbf{e}_l^H \mathbf{d}_{i,\,K}(n) + \sum_{j \neq i}^{P} \mathbf{H}_j \mathbf{d}_{j,\,K}(n) + \mathbf{v}_{i,\,K}(n)}_{\overset{\text{def}}{=} \mathbf{p}(n)}, \end{aligned} \tag{18}$$

where $d_i(n-K+1)$, as seen from (12), is the signal of interest, and $\mathbf{r}_{i,\,K}$ represents the channel response up to order K. $\mathbf{p}(n)$ denotes the ICI and MUI plus the additive noise that needs to be suppressed.

$$\mathbf{H}_i \overset{\text{def}}{=} \underbrace{\begin{bmatrix} \mathbf{h}_i(L_c-1) & \mathbf{h}_i(L_c-2) & \cdots & \mathbf{h}_i(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_i(L_c-1) & \mathbf{h}_i(L_c-2) & \cdots & \mathbf{h}_i(0) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_i(L_c-1) & \mathbf{h}_i(L_c-2) & \cdots & \mathbf{h}_i(0) \end{bmatrix}}_{K+L_c-1 \text{ blocks}} \tag{14}$$

Proposition 2: The MSE's corresponding to the coherent combiner and the MMSE equalizer are given by

$$\text{MSE}_{cc} = \frac{\mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}, \tag{19}$$

$$\text{MMSE} = \frac{1}{1 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}}^{-1} \mathbf{r}_{i,K}} \tag{20}$$

where $\mathbf{R}_{\mathbf{p},\mathbf{p}} = E\{\mathbf{p}(n)\mathbf{p}^H(n)\}$.

Proof: See Appendix A.

The MMSE in (20) sets the performance bound for all linear FIR equalizers. To evaluate the performance of the coherent combiner, consider

$$\begin{aligned}\frac{\text{MSE}_{cc}}{\text{MMSE}} &= \frac{\left(1 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}}^{-1} \mathbf{r}_{i,K}\right) \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}} \\ &= \frac{\|\mathbf{R}_{\mathbf{p},\mathbf{p}}^{-(1/2)} \mathbf{r}_{i,K}\|^2 \|\mathbf{R}_{\mathbf{p},\mathbf{p}}^{1/2} \mathbf{r}_{i,K}\|^2 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}} \\ &\geq \frac{\|\mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}}^{-(1/2)} \mathbf{R}_{\mathbf{p},\mathbf{p}}^{1/2} \mathbf{r}_{i,K}\|^2 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}} \\ &= 1\end{aligned} \tag{21}$$

with equality only when $\mathbf{R}_{\mathbf{p},\mathbf{p}}^{-(1/2)} \mathbf{r}_{i,K} = \alpha \mathbf{R}_{\mathbf{p},\mathbf{p}}^{1/2} \mathbf{r}_{i,K}$. In the conventional spatial beamforming problem, the covariance matrix of the array outputs reduces to *scalar* $\times \mathbf{I}$ when there is a large number of users uniformly distributed in space. In this scenario, the single user beamformer behaves like the optimum beamformer. Such is not the case in a frequency-selective fading environment. For 2-D RAKE receivers, $\mathbf{R}_{\mathbf{p},\mathbf{p}} = \mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K} - \mathbf{r}_{i,K}\mathbf{r}_{i,K}^H$, and

$$\mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K} = \begin{bmatrix} \mathbf{R}_{\mathbf{y},\mathbf{y}}(0) & \mathbf{R}_{\mathbf{y},\mathbf{y}}(1) & \cdots & \mathbf{R}_{\mathbf{y},\mathbf{y}}(K) \\ \mathbf{R}_{\mathbf{y},\mathbf{y}}^H(1) & \mathbf{R}_{\mathbf{y},\mathbf{y}}(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{R}_{\mathbf{y},\mathbf{y}}(1) \\ \mathbf{R}_{\mathbf{y},\mathbf{y}}^H(K) & \cdots & \mathbf{R}_{\mathbf{y},\mathbf{y}}^H(1) & \mathbf{R}_{\mathbf{y},\mathbf{y}}(0) \end{bmatrix} \tag{22}$$

is the autocovariance matrix of the vector sequence $\mathbf{y}(n)$: $\mathbf{R}_{\mathbf{y},\mathbf{y}}(l) = E\{\mathbf{y}(n)\mathbf{y}^H(n-l)\}$. Each submatrix $\mathbf{R}_{\mathbf{y},\mathbf{y}}(l)$ may become *scalar* $\times \mathbf{I}$ in the presence of a large number of users, and the overall matrix $\mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K}$ is always temporally colored due to the channel effects. In other words, the performance of the coherent combiner is always interior to that of the MMSE equalizer. Such a limitation is inherent and cannot be overcome using spatial processing. Equalizers must be applied to achieve optimum performance.

Proposition 3: In a frequency-selective fading situation, the MSE of the coherent combiner output is strictly larger than the MMSE regardless of the spatial distribution of the users.

IV. MMSE Equalizer Estimation

In this section, we consider the estimation of the MMSE equalizer from the multichannel outputs without knowing the inputs. Given the parametric model in (13) and a finite number of observations, one can, in principle, obtain channel and noise parameter estimates via maximum likelihood (ML) estimation and then construct the MMSE equalizers accordingly based on the expression given in (17) [25]. However, receiver design through optimum channel estimation is computationally expensive and, more importantly, may be susceptible to model mismatch [22], [23]. From a practical viewpoint, it would be of great interest to find robust methods that yield satisfactory performance with affordable cost.

The explicit expression of the MMSE equalizers derived in the previous section suggests that a direct estimation procedure may be developed. Indeed, as seen from (17), one only needs to estimate $\mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K}$, which can obviously be approximated with the data covariance matrix, and vector $\mathbf{r}_{i,K}$ to calculate $\mathbf{g}_{i,mmse}$. In the following, we derive a principal component algorithm to determine $\mathbf{r}_{i,K}$ from $\{\mathbf{y}(n)\}$.

A. The Principal Component Algorithm

The foundation to the principle component method, which was initially formulated for array response vector estimation for CDMA systems in a frequency nonselective environment [26], is the observation that the pre- and post-despreading covariance matrices of antenna outputs are different only by a rank one matrix defined by $\mathbf{r}_{i,K}$. Therefore, $\mathbf{r}_{i,K}$ can be estimated as the principal eigenvector of the difference matrix. Similar approaches have been suggested to estimate $\{\mathbf{a}_i(l)\}$ in (1) when the multipath channels are frequency selective [15]. However, their performance hinges on the accurate estimation of multipath delays. By dealing with composite channels rather than individual multipath signals, no restrictive assumptions are required in the present discrete-time framework.

From (18) and Assumptions A1) and A2), we observe that

$$\mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K} = \mathbf{r}_{i,K}\mathbf{r}_{i,K}^H + \mathbf{R}_{\mathbf{p},\mathbf{p}}. \tag{23}$$

Consequently, $\mathbf{R}_{\mathbf{Y}_i} = E\{\mathbf{Y}_i(k)\mathbf{Y}_i^H(k)\} = L\mathbf{R}_{\mathbf{y}_K,\mathbf{y}_K} = L\mathbf{r}_i\mathbf{r}_i^H + L\mathbf{R}_{\mathbf{p},\mathbf{p}}$.

By despreading the antenna outputs, the power of the desired signal is enhanced by a factor of L^2, whereas the power of interference increases linearly by a factor of L. Using (10) and (18),

$$\begin{aligned}\mathbf{x}_{i,K}(k) &= \mathbf{Y}_i(k)\mathbf{c}_i(k) \\ &= Ls_i(k)\mathbf{r}_{i,K} \\ &\quad + \sum_{l=0}^{L-1} \mathbf{p}(kL + n_i + K + l - 1)\mathbf{c}_i(k, l).\end{aligned} \tag{24}$$

Therefore under Assumptions A1)–A3)

$$\begin{aligned}\mathbf{R}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} &= E\{\mathbf{x}_{i,K}, \mathbf{x}_{i,K}^H\} \\ &= L^2\mathbf{r}_{i,K}\mathbf{r}_{i,K}^H + L\mathbf{R}_{\mathbf{p},\mathbf{p}}.\end{aligned} \tag{25}$$

Comparing the above covariance matrix with $\mathbf{R}_{\mathbf{Y}_i}$, it is readily seen that

$$\mathbf{R}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} - \mathbf{R}_{\mathbf{Y}_i} = L(L-1)\mathbf{r}_{i,K}\mathbf{r}_{i,K}^H. \tag{26}$$

The above result indicates that the span of $\mathbf{R}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} - \mathbf{R}_{\mathbf{Y}_i}$ is defined by vector $\mathbf{r}_{i,K}$. Hence, $\mathbf{r}_{i,K}$ can be determined as the principal eigenvector of $\mathbf{R}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} - \mathbf{R}_{\mathbf{Y}_i}$. In practice,

$\mathbf{R}_{\mathbf{x}_i,\mathbf{x}_i} - \mathbf{R}_{\mathbf{Y}_i}$ is unknown and is therefore estimated by the sample correlation matrix:

$$\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} - \hat{\mathbf{R}}_{\mathbf{Y}_i} = \frac{1}{N}\left[\sum_{k=1}^{N}\mathbf{Y}_i(k)\mathbf{c}_i(k)\mathbf{c}_i^H(k)\mathbf{Y}_i^H(k) - \sum_{k=1}^{N}\mathbf{Y}_i(k)\mathbf{Y}_i^H(k)\right]. \tag{27}$$

Notice that $\mathbf{r}_{i,K} = [\mathbf{h}_i^H(0)\cdots\mathbf{h}_i^H(L_c-1), \mathbf{0}\cdots\mathbf{0}]^H$, and the above method also provides an effective way to estimate the multipath channel $\{\mathbf{h}_i(l), l=0,\cdots,L_c-1\}$. Compared with the matched filter outputs that are generally biased estimates of the channel responses due to ICI, the channel estimates given by the principal component method are certainly more accurate. The principal eigenvector of $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} - \hat{\mathbf{R}}_{\mathbf{Y}_i}$ can thus be used in a coherent combiner to increase its performance.

Once we have the $\mathbf{R}_{\mathbf{y}_{i,K},\mathbf{y}_{i,K}}$ and $\mathbf{r}_{i,K}$ estimates, the MMSE equalizer can be computed accordingly. The following summarizes the estimate procedure:

1) Calculate the data covariance matrices as

$$\hat{\mathbf{R}}_{\mathbf{Y}_i} = \frac{1}{N}\left[\sum_{k=1}^{N}\mathbf{Y}_i(k)\mathbf{Y}_i^H(k)\right], \tag{28}$$

$$\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}} = \frac{1}{N}\left[\sum_{k=1}^{N}\mathbf{Y}_i(k)\mathbf{c}_i(k)\mathbf{c}_i^H(k)\mathbf{Y}_i^H(k)\right]. \tag{29}$$

2) Estimate $\hat{\mathbf{r}}_{i,K}$ as the principal eigenvector of $\hat{\mathbf{R}}_{\mathbf{Y}_i} - \hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$.
3) The MMSE equalizer within a scalar multiple is given by

$$\hat{\mathbf{g}}_{i,mmse} = \hat{\mathbf{R}}_{\mathbf{Y}_i}^{-1}\hat{\mathbf{r}}_{i,K}, \qquad i=1,\cdots,P. \tag{30}$$

Theoretically, the above approach provides unbiased estimates of the MMSE equalizer given sufficient data samples. However, since $\mathbf{r}_{i,K}$ is estimated one-by-one, as opposed to the optimum ML, the principal component method may not be statistically efficient. Nevertheless, it possesses the robustness and simplicity essential to practical implementation. In the remainder of this paper, we shall refer to the MMSE equalizer estimate given in (30) as the principal component (PC) MMSE equalizer.

B. Comparison with Postdespreading 2-D RAKE Receivers

This algorithm is similar to the 2-D RAKE receiver proposed by Zoltowski *et al.* in [27] and [28]. The algorithm of Zoltowski *et al.* is premised on despreading at a large number of delay times on the order of L, which is the number of chips per bit, as opposed to K, as in the PC MMSE algorithm proposed above. In this case, as we despread based on the ith user's code, the ith user's contribution to the resulting output is essentially a couple of fingers per bit (one per multipath arrival) lying over a discrete-time interval of duration K commensurate with the multipath time delay spread. Away from the RAKE finger peaks, the ith user's contribution to the despreaded output is reduced by a factor of L.

In the algorithm of Zoltowski *et al.*, $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$ is formed exactly the same as in (29), but $\hat{\mathbf{R}}_{\mathbf{Y}_i}$ is formed by sliding a time window of duration K behind of each of the N antennas over the complementary portion of the postdespreading bit interval away from where $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$ is formed. Based on a maximization of SINR principle, Zoltowski *et al.* propose computing $\hat{\mathbf{g}}_{i,mmse}$ as the principal generalized eigenvector of the matrix pencil $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}\{\hat{\mathbf{R}}_{\mathbf{Y}_i}\}$. However, exploiting the fact that in the asymptotic case $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$ and $\hat{\mathbf{R}}_{\mathbf{Y}_i}$ differ only by a rank one contribution proportional to $\hat{\mathbf{r}}_{i,K}\hat{\mathbf{r}}_{i,K}^H$, it is easily shown that this is equivalent to estimating $\hat{\mathbf{r}}_{i,K}$ as the principal eigenvector of $\hat{\mathbf{R}}_{\mathbf{Y}_i} - \hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$ and forming $\hat{\mathbf{g}}_{i,mmse} = \hat{\mathbf{R}}_{\mathbf{Y}_i}^{-1}\hat{\mathbf{r}}_{i,K}$.

As discussed at the end of Section III (see the discussion surrounding (10) and (11)), multiplying by the code for a number of delay times is equivalent to running the received data at each antenna through an FIR filter whose coefficients are the chip values $c_i(k;l), l=1,\cdots,L$. The PC MMSE algorithm proposed above assumes that the spatio-temporal characteristics of the other $P-1$ users are the same before and after FIR filtering by this code so that $\hat{\mathbf{R}}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}$ and $\hat{\mathbf{R}}_{\mathbf{Y}_i}$ asymptotically differ only by a rank one contribution due to the ith user. This assumption is generally true with chip rate sampling if the chip waveform is rectangular. Under these conditions and Assumptions A2) and A3), the sampled signal prior to despreading for each user is i.i.d. In addition, A3) implies that the frequency response of the FIR filter based on the ith user's code is approximately all pass, i.e., flat. Thus, each user is effectively directional white noise before and after FIR filtering with the code, i.e., pre and postdespreading. The assumption of a rectangular waveform can be relaxed, assuming strict synchronization, and that the chip waveform is zero at kT, $k \neq 0$, as is the case using a raised cosine spectrum.

However, if one samples greater than the chip rate, the temporal characteristics of the other $P-1$ users and the receiver noise are generally altered by passing the signal through an FIR filter based on commensurate sampling of the spreading waveform of the ith user. For example, consider sampling two times per chip; this provides a robustness to synchronization and increases the probability of sampling at the multipath arrival times as required by the ideal 2-D RAKE receiver. The frequency response of the FIR filter obtained by sampling the spreading waveform of the desired user at twice the chip rate is no longer flat. This negates the efficacy of the PC MMSE algorithm. However, the algorithm of Zoltowski *et al.* is still applicable since both matrices are formed postdespreading.

Recall, though, that this algorithm requires despreading at a large number of delay times—on the order of L, which is the number of chips per bit—as opposed to K, as in the PC MMSE algorithm. Each output point requires L multiples and $L-1$ additions. As the fingers generally occupy a small fraction of a bit interval, the computational load of the Zoltowski *et al.* algorithm is much greater than the PC MMSE algorithm.

Thus, if one has achieved fairly accurate synchronization, the PC MMSE algorithm is an attractive 2-D RAKE receiver from both a performance and computation point of view.

C. Equivalence to Symbol Sequence Optimization

Toward this stage, our derivation has been based on the minimization of the MSE of the desired chip sequence rather than the symbol sequence that actually bears information. From a reception viewpoint, it is apparently more plausible to adopt the MSE of symbol estimates $E\{|\hat{s}_i(n) - s_i(n)|^2\}$ as the optimization criterion. In this section, we show that the MMSE equalizer given in (17) also minimizes the MSE of symbol estimates. In other words, chip-level optimization is equivalent to symbol-level optimization.

The MMSE equalizer for symbol estimates is found by minimizing

$$E\{|\mathbf{g}_i\mathbf{x}_{i,K}(k) - s_i(k)|^2\} = E\{|\mathbf{g}_i\mathbf{Y}_i(k)\mathbf{c}_i(k) - s_i(k)|^2\} \tag{31}$$

with respect to $\mathbf{g}_i$. Taking the derivative of the above expression and setting it to zero, we have, because of (18) and (25)

$$\begin{aligned}\mathbf{g}_i &= \mathbf{R}^{-1}_{\mathbf{x}_{i,K},\mathbf{x}_{i,K}}\mathbf{r}_{i,K}\\ &= \frac{1}{L}(L\mathbf{r}_{i,K}\mathbf{r}^H_{i,K} + \mathbf{R}_{\mathbf{p},\mathbf{p}})^{-1}\mathbf{r}_{i,K}.\end{aligned} \tag{32}$$

Recall from (17) that $\mathbf{g}_{i,mmse} = \mathbf{R}^{-1}_{\mathbf{y}_K,\mathbf{y}_K}\mathbf{r}_{i,K} = (\mathbf{r}_i\mathbf{r}_i^H + \mathbf{R}_{\mathbf{p},\mathbf{p}})^{-1}\mathbf{r}_{i,K}$. The following easily proved lemma establishes the correspondence between $\mathbf{g}_{i,mmse}$ and $\mathbf{g}_i$ in the above equation.

Lemma 1: Given an invertible matrix $\mathbf{R}$ and a column vector $\mathbf{r}$, $(\mathbf{R} + \mathbf{r}\mathbf{r}^H)^{-1}\mathbf{r} = \alpha\mathbf{R}^{-1}\mathbf{r}$ where $\alpha = 1 + \mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}$.

Proof: Using the matrix inversion lemma,

$$\begin{aligned}(\mathbf{R} + \mathbf{r}\mathbf{r}^H)^{-1}\mathbf{r} &= \left(\mathbf{R}^{-1} - \frac{\mathbf{R}^{-1}\mathbf{r}\mathbf{r}^H\mathbf{R}^{-1}}{1 + \mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}}\right)\mathbf{r}\\ &= \mathbf{R}^{-1}\mathbf{r} - \frac{(\mathbf{r}^H\mathbf{R}^{-1}\mathbf{r})\mathbf{R}^{-1}\mathbf{r}}{1 + \mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}}\\ &= \frac{\mathbf{R}^{-1}\mathbf{r}}{1 + \mathbf{r}^H\mathbf{R}^{-1}\mathbf{r}}.\end{aligned} \tag{33}$$

□

Therefore, $\mathbf{g}_{i,mmse}$ and $\mathbf{g}_i$ in (32) are identical within a scalar multiple.

Proposition 4: The equalizer that minimizes the MSE of chip sequence estimates also minimizes the MSE of symbol sequence estimates.

Following the derivation in Appendix A, it is straightforward to derive the following result.

Proposition 5: The MMSE of the ith symbol sequence estimates using a vector linear equalizer of order K is given by

$$\text{MMSE}_s = \frac{1}{1 + L\mathbf{r}^H_{i,K}\mathbf{R}_{\mathbf{p},\mathbf{p}}\mathbf{r}_{i,K}}. \tag{34}$$

D. Order Selection

One important practical issue in 2-D RAKE receivers is the selection of the length of the equalizer K. Intuitively, the order of the receiver should be at least L_c so that a multipath signal with a maximum delay can be incorporated. The question, however, is whether or not a longer equalizer can offer better performance. The proposition below provides insight to this question.

Proposition 6: Let MMSE_K be the MMSE of chip sequence estimates using linear equalizer of order K. Then

$$\text{MMSE}_K \geq \text{MMSE}_{K+1}$$

when $K \geq L_c$.

Proof: The proof is simple and is thus omitted. □

The implication of the above proposition is that a longer equalizer is generally preferred if it is affordable. On the other hand, numerical evaluation of (20) in Section VI reveals that after K reaches L_c, the theoretic MMSE reduction due to additional taps is noticeable only when $P < M$. In most scenarios, a 2-D RAKE receiver approaches its full potential when the order of the equalizer reaches L_c.

V. Deterministic Blind Equalization

One of the limitations of the PC method is its reliance on the covariance matrices of antenna outputs. In a rapidly varying environment, the available data may not be adequate for statistics-based approaches to yield satisfactory parameter estimates. The finite data effect is inherent and is difficult to overcome unless sufficient information of the underlying data structure is available. Signal estimation without the use of statistical information is often termed *deterministic* blind equalization and has been successfully used in many areas [19].

Our goal in this section will be to find data efficient deterministic solutions of 2-D RAKE receivers. We are particularly interested in the so-called *zero-forcing* equalizers for an underloaded CDMA system, where the number of effective channels is greater than the number of users. In this scenario, the degree of freedom of outputs becomes larger than that of the inputs, and it is then feasible to perfectly recover the transmitted signals in the absence of noise (zero forcing). The method presented here builds on an almost trivial observation, i.e., knowledge of the PN spreading sequences can sidestep the use of training sequences and allows us to formulate a least-squares problem to determine the vector equalizer.

Ignore the noise term, and rewrite (13) as

$$\mathbf{y}_K(n) = \underbrace{[\mathbf{H}_1 \quad \cdots \quad \mathbf{H}_P]}_{\stackrel{\text{def}}{=}\mathbf{H}}\left[\mathbf{d}^H_{1,K}(n) \cdots \mathbf{d}^H_{P,K}(n)\right]^H.$$

It is seen that the dimension of the channel matrix $\mathbf{H}$ is $MK \times P(L_c + K - 1)$. Therefore, when $M > P$, there exists a $K \geq P(L_c - 1)/(M - P)$ such that $\mathbf{H}$ has more rows than columns and, thus, is of full column rank in general. The possibility of zero forcing the receiver arises. All signals can be separated by applying the pseudo-inverse of $\mathbf{H}$, $\mathbf{H}^\dagger$, to $\mathbf{y}_K(n)$

$$\begin{aligned}\mathbf{H}^\dagger \mathbf{y}_K(n) &= \mathbf{H}^\dagger \mathbf{H}[\mathbf{d}_{1,K}^H(n) \cdots \mathbf{d}_{P,K}^H(n)]^H \\ &= [\mathbf{d}_{1,K}^H(n) \cdots \mathbf{d}_{P,K}^H(n)]^H.\end{aligned}$$

For more discussions on zero-forcing equalization, see [23] and references therein. Note that without incorporating the temporal diversity, complete nulling of interference using spatial beamforming requires the number of receivers to be larger than the *total* number of multipath reflections [29].

When $M > P$, the MMSE equalizer given in (17) certainly qualifies as a zero-forcing equalizer in the absence of noise. To see this, replace $\mathbf{R}_{\mathbf{y}_K, \mathbf{y}_K}^{-1}$ in (17) with its pseudo-inverse, and consider the MMSE equalizer for the first user:

$$\begin{aligned}\mathbf{g}_{1,mmse} &= (\mathbf{H}\mathbf{H}^H)^\dagger \mathbf{r}_{1,K} \\ &= (\mathbf{H}\mathbf{H}^H)^\dagger \mathbf{H}\mathbf{e}_L \\ &= (\mathbf{H}\mathbf{H}^H)^\dagger \mathbf{H} \underbrace{\mathbf{H}^H\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}}_{=\mathbf{I}} \mathbf{e}_L \\ &= \mathbf{H}^\dagger \mathbf{e}_L. \end{aligned} \tag{35}$$

$\mathbf{g}_{1,mmse}$ is zero forcing since $\mathbf{g}_{1,mmse}^H \mathbf{y}_K(n) = d_1(n-K+1)$. However, as we pointed out at the begin of this section, $\hat{\mathbf{g}}_{1,mmse}$ given by the PC method may not be exact due to the finite data effect. To find the deterministic solution, assuming $\mathbf{g}_i$ is a zero-forcing equalizer for the ith user,

$$\begin{aligned}\mathbf{g}_i^H \mathbf{Y}_i(k) &= [d_i(kL+n_i), \cdots, d_i(kL+n_i+L-1)] \\ &= s_i(k)\mathbf{c}_i^H(k). \end{aligned} \tag{36}$$

The only unknowns in the above equation are $\mathbf{g}_i$ and $\{s_i(n)\}$. We may thus formulate a minimization problem with respect to $\mathbf{g}_i(n)$ and $s_i(1), s_i(2), \cdots, s_i(N)$, given a finite number of observations $\mathbf{Y}_i(k)$, $k = 1, 2, \cdots, N$:

$$\begin{aligned}\{\hat{\mathbf{g}}, \hat{s}_i(1), \cdots, \hat{s}_i(N)\} &= \arg \min_{\mathbf{g}, s_i(1), \cdots, s_i(N)} \sum_{k=1}^N \|\mathbf{g}^H \mathbf{Y}_i(k) \\ &\quad - s_i(k)\mathbf{c}_i^H(k)\|^2. \end{aligned} \tag{37}$$

Representing the above minimization problem in matrix form,

$$\begin{bmatrix} \mathbf{Y}_i^H(1) & -\mathbf{c}_i(1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{Y}_i^H(2) & \mathbf{0} & -\mathbf{c}_i(2) & \cdots & \mathbf{0} \\ \vdots & \vdots & \mathbf{0} & \ddots & \vdots \\ \mathbf{Y}_i^H(N) & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{c}_i(N) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{g}_i \\ s_i(1) \\ \vdots \\ s_i(N) \end{bmatrix} = \mathbf{0} \tag{38}$$

it is seen that there are NL equations and $MK+N$ unknowns, which evidently defines an overdetermined least-squares problem when $N \geq (MK)/(L-1)$. Therefore, the vector equalizer $\mathbf{g}_i$ can be determined from the nontrivial solution of (38). Since the parameters of interest are the coefficients in $\mathbf{g}_i$, structures and redundancy can be exploited to simplify the solution.

Proposition 7: Given $\{\mathbf{Y}_i(k)\}_{i=1}^N$, $\mathbf{g}_i$ that satisfies (38) is found as the least significant eigenvector of $(\hat{\mathbf{R}}_\mathbf{Y} - \hat{\mathbf{R}}_\mathbf{x}/L)$, where $\hat{\mathbf{R}}_\mathbf{Y}$ and $\hat{\mathbf{R}}_\mathbf{x}$ are defined in (28) and (29), respectively.

Proof: See Appendix B. □

To verify that the above solution is indeed zero forcing, decompose $\mathbf{Y}_i(k)$ into

$$\mathbf{Y}_i(k) = s_i(k)\mathbf{r}_{i,K}\mathbf{c}_i^H(k) + \tilde{\mathbf{H}}\mathbf{P}(k)$$

where $\tilde{\mathbf{H}}$ is the $\mathbf{H}$ matrix without vector $\mathbf{r}_{i,K}$, and $\mathbf{P}(k)$ denotes the interfering signals. Notice that

$$\begin{aligned}\mathbf{Y}_i(k)\mathbf{Y}_i(k) &= L\mathbf{r}_{i,K}\mathbf{r}_{i,K}^H + 2\Re[s_i(k)\mathbf{r}_{i,K}\mathbf{c}_i^H(k)\mathbf{P}(k)\tilde{\mathbf{H}}^H] \\ &\quad + \tilde{\mathbf{H}}\mathbf{P}(k)\mathbf{P}^H(k)\tilde{\mathbf{H}}^H. \end{aligned} \tag{39}$$

Using

$$\begin{aligned}\mathbf{x}_{i,K}(k) &= \mathbf{Y}_i(k)\mathbf{c}_i(k) \\ &= Ls_i(k)\mathbf{r}_{i,K} + \tilde{\mathbf{H}}\mathbf{P}(k)\mathbf{c}_i(k)\end{aligned}$$

it is easily shown that

$$\begin{aligned}&\mathbf{x}_{i,K}(k)\mathbf{x}_{i,K}^H(k) \\ &\quad = L\{L\mathbf{r}_{i,K}\mathbf{r}_{i,K}^H + 2\Re[s_i(k)\mathbf{r}_{i,K}\mathbf{c}_i^H\mathbf{P}(k)\tilde{\mathbf{H}}^H]\} \\ &\qquad + \tilde{\mathbf{H}}\mathbf{P}(k)\mathbf{c}_i(k)\mathbf{c}_i^H(k)\mathbf{P}^H(k)\tilde{\mathbf{H}}^H. \end{aligned} \tag{40}$$

Consequently,

$$\begin{aligned}\hat{\mathbf{R}}_\mathbf{Y} - \frac{\hat{\mathbf{R}}_\mathbf{x}}{L} &= \tilde{\mathbf{H}}\left[\sum_{k=1}^N \mathbf{P}(k)\mathbf{P}^H(k)\right. \\ &\quad \left. - \sum_{k=1}^N \mathbf{P}(k)\mathbf{c}_i(k)\mathbf{c}_i^H(k)\mathbf{P}^H(k)\right]\tilde{\mathbf{H}}^H.\end{aligned}$$

The column span of the above matrix is defined by the interfering channel matrix $\tilde{\mathbf{H}}$, which has at least one null vector under the assumption that $\mathbf{H}$ is of full column rank. An equalizer is zero forcing if

$$\mathbf{g}_i^H\mathbf{r}_{i,K} = \alpha \neq 0, \mathbf{g}_i^H\tilde{\mathbf{H}} = \mathbf{0}.$$

The least significant null vector of $\hat{\mathbf{R}}_\mathbf{Y} - \hat{\mathbf{R}}_\mathbf{x}/L$ is orthogonal to $\tilde{\mathbf{H}}$ but not to $\mathbf{r}_{i,K}$ since $\mathbf{r}_{i,K}$ is linearly independent with $\tilde{\mathbf{H}}$ and thus qualifies as a zero-forcing equalizer.

The least significant equalizer is optimum in a deterministic sense in interference suppression even if the zero-forcing condition is not satisfied. Therefore, the deterministic method is also applicable to systems with $P > M$. It performance, however, is difficult to predict without a full-scale performance analysis. The most suitable application of the least-squares method is probably in underloaded CDMA systems with a high SNR. Otherwise, the PC MMSE equalizer that accounts for the statistics of the input signals is preferable, provided that a reasonable number of data samples is available.

Before moving on to the next section, we note the following:

- Unlike the case when $P > M$, the length of the equalizer plays a critical role in underloaded systems. Theoretically, a zero-forcing equalizer exists only if $K \geq P(L_c-1)/(M-P)$. This implies that before K reaches $P(L_c-1)/(M-P)$, the performance of the 2-D RAKE receiver can be increased by using longer equalizers.

- By identifying the zero-forcing equalizer as the least significant eigenvector, knowledge of neither the channel length nor the statistics of the inputs is required. On the other hand, the least-squares approach does require N to be greater than $(MK)/(L-1)$ to guarantee a unique solution. Remember that K must be greater than $P(L_c-1)/(M-P)$ to satisfy the zero-forcing condition. Hence, the minimum N for identifying an exact zero-forcing equalizer is $MP(L_c-1)/(M-P)(L-1)$.

VI. Numerical Results

In this section, we examine the behavior and performance of the proposed blind equalization algorithms using computer simulations. We adopt the output signal-to-interference ratio (SIR) of symbol estimates as the performance measure and calculate it as

$$\text{SIR} = -10\,\log_{10}\,\text{MSE}\,[\text{dB}].$$

All simulation cases here involve an eight-element antenna array and CDMA signals with a spreading factor of 32. Temporal oversampling was not used (i.e., chip rate sampling). The PN sequences were randomly generated rather than created using shift registers. Each composite channel comprises direct path and multipath components—the delays and the number of multipath components were chosen uniformly from $[0 \quad 3T]$ and $[1 \quad 10]$, respectively. The maximum channel length is, thus, 3. The raised-cosine function with a roll-off factor of 0.5 was used, and the SNR was set at 20 dB. A total of 200 Monte-Carlo runs were done for each simulation. We assumed mild power control in the system so that the power variations among users were within 6 dB. The transmitted power of each user is defined as the total power received by the antenna array:

$$P_i = \sum_{l=0}^{L_c-1} ||\mathbf{h}_i(l)||^2, \qquad i = 1, \cdots, P. \tag{41}$$

The first case involved 15 users. Data samples within 10–60 symbol periods were used for MMSE equalizer estimation. For comparison, the channel estimate obtained from $\hat{\mathbf{r}}_{i,K}$ was used in the coherent combiner. Fig. 3 plots the output SIR's corresponding to the coherent combiner and the proposed PC MMSE equalizer for one user. The superiority of the proposed equalizer is clearly demonstrated. Note that as the number of samples increases, the performance of the PC method approaches the optimum upper bound: $-10\,\log_{10}\,\text{MMSE}_i$, which suggests the proposed method is nearly optimum given sufficient data. On the other hand, the noticeable gap between the upper bound and the simulation SIR's implies that the PC estimator is not statistically efficient.

In Fig. 4, the difference between the simulation SIR's and their theoretic upper bound are plotted for all users in the systems. The top three lines, which correspond to the PC method using samples within 30, 45, and 60 symbol periods, are close to zero for all users. This demonstrates that the proposed estimation method is near-far resistant—a feature highly desirable in CDMA communications. At the same time, it is not surprising to see that the bottom three lines, which correspond to the coherent combiner, have large variations due to imperfect power control, even through the channel estimates from the PC method were satisfactory.

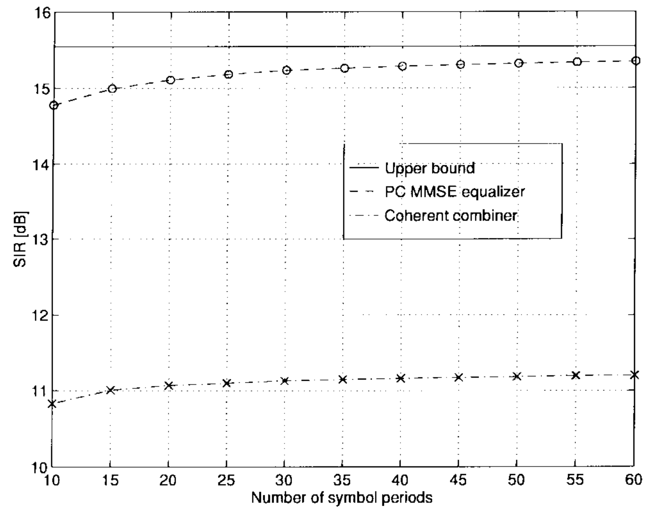

Fig. 3. SIR versus number of samples.

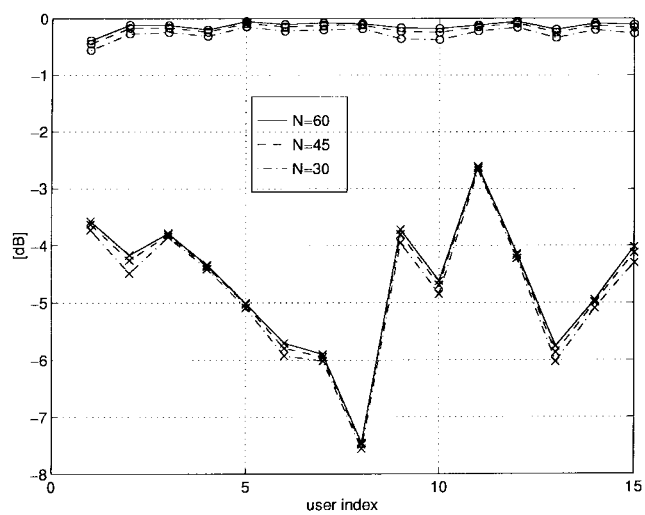

Fig. 4. Simulation SIR's minus optimum SIR's.

In the second case, we fixed N at 30 and varied the number of users to study how variations of MUI affect the output SIR of a desired user. The results are plotted Fig. 5. As expected, the SIR's decrease as the number of users increases. In addition, we observe that there is a consistent gap between the performance of the PC MMSE equalizer and that of the coherent combiner. This verifies the assertion of Proposition 3 that the MMSE equalizer always outperforms the coherent combiner in a frequency-selective environment, regardless of the number of active users.

The theoretic SIR of the MMSE equalizer versus the filter length are plotted in Fig. 6 for different P values. In all cases, a longer equalizer yields a higher SIR. However, the performance improvement due to an additional tap is evident only when K is small. After K reaches L_c, the SIR's become saturated in most scenarios except for $P=6$, where the SIR continues to increase until $K=6$. The results agree with our discussions in Section V. In an underloaded system, the SIR

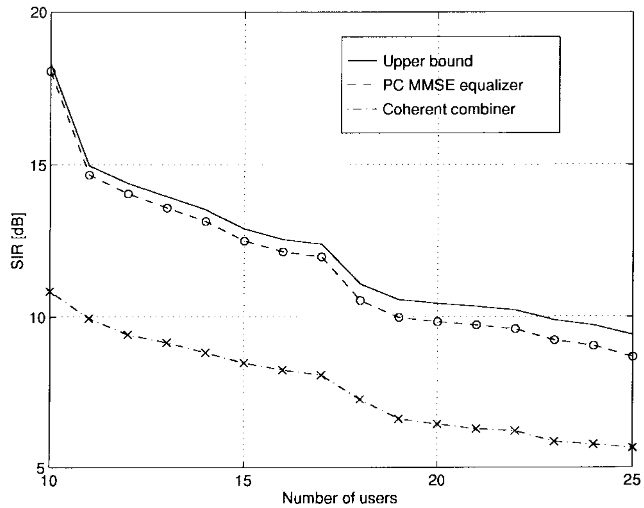

Fig. 5. SIR versus number of users.

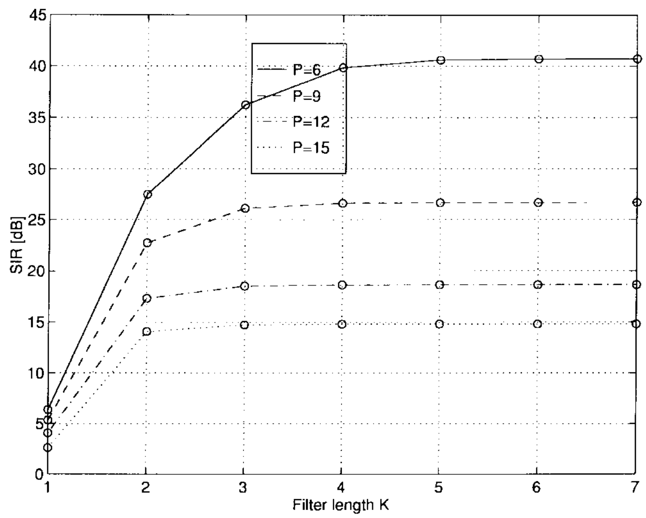

Fig. 6. SIR versus filter length.

of the MMSE equalizer output increases until the zero-forcing condition is satisfied. In this particular case, the minimum filter length required to accomplish zero-forcing equalization is $K \geq P(L_c - 1)/(M - P) = 6$.

The last example serves to compare the performance of different approaches in an underloaded systems with SNR at 40 dB. The filter length was fixed at $K = 3$. Data samples within 10 symbol periods were used in both the least-squares and the PC methods. The performance comparison is plotted in Fig. 7. When the number of users is small, the least-squares method shows its advantages in interference suppression. Note that with $K = 3$, one can, in principle, null out signals from three cochannel users. After the total number of users reaches 4, the zero-forcing condition no longer holds, and the performance of the least-squares approach drops below that of the PC MMSE equalizer. It is interesting to notice that unlike the coherent combiner and the PC MMSE equalizer, the output SIR corresponding to the least-squares method does not decrease monotonically with the number of users. This is explainable, considering the fact that $\mathbf{r}_{i,K}$ of the desired signal is not incorporated in the least-squares approach. It is possible

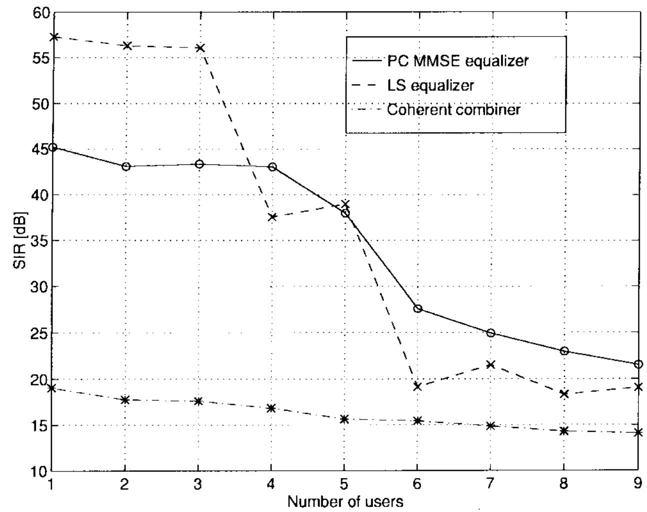

Fig. 7. SIR's in an underloaded system.

that with an additional user, the resulting equalizer can better accommodate the desired signal, thus effectively increasing the SIR.

VII. Conclusion

Conventional 2-D RAKE receivers that premise on accurate estimation of multipath delays have inherent difficulties in a rich multipath environment. By simultaneously exploiting the spatial and temporal diversities in discrete channels, we have formulated in this paper the optimum linear FIR equalizer without placing any constraints on the multipath structure. A principal component method has been developed to estimate the MMSE equalizer. The new algorithm offers nearly optimum signal estimates and possesses the simplicity essential to practical implementation. A data efficient least-squares approach that can accomplish blind equalizer estimation without requiring any statistical knowledge of the inputs is also presented.

Appendix A
MSE Calculation

The MMSE can be calculated by substituting (17) into

$$\text{MMSE} = E\{|\mathbf{g}_{i,mmse}^H \mathbf{y}_K(n) - d_i(n-K+1)|^2\}. \quad (42)$$

With straightforward manipulation,

$$\begin{aligned}\text{MMSE} &= 1 - \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{y}_K\mathbf{y}_K}^{-1} \mathbf{r}_{i,K} \quad (43)\\ &= 1 - \mathbf{r}_{i,K}^H (\mathbf{r}_{i,K}\mathbf{r}_{i,K}^H + \mathbf{R}_{\mathbf{p},\mathbf{p}})^{-1} \mathbf{r}_{i,K}^H. \quad (44)\end{aligned}$$

Applying the matrix inversion lemma yields

$$\text{MMSE} = \frac{1}{1 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}}^{-1} \mathbf{r}_{i,K}}. \quad (45)$$

Similarly, for the coherent combining approach, where $\mathbf{g}_{i,cc} = \alpha \mathbf{r}_{i,K}$, we have

$$\begin{aligned}\text{MSE}_{cc} &= E\{|\alpha \mathbf{r}_{i,K}^H \mathbf{y}_K(n) - d_i(n-K+1)|^2\}\\ &= \alpha^2(\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K})\\ &\quad - 2\alpha\|\mathbf{r}_{i,K}\|^2 + 1. \quad (46)\end{aligned}$$

Taking the derivative of MEE_{cc} with respect to α and setting it to zero,

$$\alpha = \frac{\|\mathbf{r}_{i,K}\|^2}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}.$$

Substituting the above result back to (46), we obtain

$$\mathrm{MSE}_{cc} = \frac{\mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}{\|\mathbf{r}_{i,K}\|^4 + \mathbf{r}_{i,K}^H \mathbf{R}_{\mathbf{p},\mathbf{p}} \mathbf{r}_{i,K}}.$$

APPENDIX B
PROOF OF PROPOSITION 7

Rewrite (38) as

$$\begin{bmatrix} \mathbf{Y}_i^H(1) \\ \vdots \\ \mathbf{Y}_i^H(N) \end{bmatrix} \mathbf{g}_i = \begin{bmatrix} \mathbf{c}_i(1) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{c}_i(N) \end{bmatrix} \begin{bmatrix} s_i(1) \\ \vdots \\ s_i(N) \end{bmatrix}. \quad (47)$$

We may express $\mathbf{s}_i \stackrel{\text{def}}{=} [s_i(1) \cdots s_i(N)]^T$ in terms of $\mathbf{g}_i$ as

$$\mathbf{s}_i = \frac{1}{L} \begin{bmatrix} \mathbf{c}_i^H(1)\mathbf{Y}_i^H(1) \\ \vdots \\ \mathbf{c}_i^H(N)\mathbf{Y}_i^H(N) \end{bmatrix} \mathbf{g}_i.$$

Substituting the above result into (47) and multiplying both sides with $[\mathbf{Y}_i(1) \cdots \mathbf{Y}_i(N)]$, we have

$$\left\{ \sum_{i=1}^{N} \left[\mathbf{Y}_i(i)\mathbf{Y}_i(i) - \frac{\mathbf{Y}_i(i)\mathbf{c}_i(k)\mathbf{c}_i^H(k)\mathbf{Y}_i(i)}{L} \right] \right\} \mathbf{g}_i = \mathbf{0}.$$

$\mathbf{g}_i$ can thus be found at the least significant eigenvector of the about matrix.

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