A Deterministic Approach to Blind Symbol Estimation

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Abstract—**A blind symbol estimation technique for digital communication is developed by exploiting a special data structure of the oversampled channel output. The proposed method achieves direct symbol estimation without determining the channel characteristics. Moreover, if the transmitting symbols belong to a finite set of alphabets, the new approach can be extended to handle multiple sources.**

I. Introduction

The problem of blind channel identification using the second-order statistics of the *oversampled* channel output has drawn considerable attention recently following the pioneering work of Tong *et al.* [8]. Many interesting results have been developed [9], [2], [1], [4]–[6]. Although the second-order approach in [8] has significant advantages over the conventional high-order methods, it is still statistics based and may suffer from finite data effect when dealing with an extremely short sample sequence. As an attempt to remedy this problem and provide an algebraic interpretation of the blind identification problem, we establish a deterministic model for the multichannel communication system by treating input sequence as an unknown deterministic signal [3]. Similar data models have been adopted by various researchers, and many data efficient parametric blind estimation approaches have been developed [3]–[6].

The focus of the aforementioned methods has been on the blind identification of the channel. In this letter, we extend our investigation and propose a new algorithm that accomplishes symbol estimation directly from the oversampled output data of the channel.

II. Problem Statement

Using the spatial and/or temporal oversampling techniques [8], a communication channel can be cast into a single-input multiple-output (SIMO) system with multiple subchannels. Denote $x_i(\cdot)$ the outputs from the ith subchannel; they are related to the FIR channel response $\{h_i(\cdot)\}$ and the system inputs $s(\cdot)$ by

$$x_i(k) = \sum_{j=0}^{L} h_i(j)s(k-j), \qquad i = 1 \cdots M, \tag{1}$$

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where M is the number of subchannels, and L is the maximum order of all subchannels. The case of principal interest herein is the estimation of the input symbols $s(\cdot)$ from *finite* data samples without the knowledge of the channel characteristics.

Let $\boldsymbol{x}_k = [x_1(k) \cdots x_M(k)]^T, \boldsymbol{h}_l = [h_1(l) \cdots h_M(l)]^T$. For a finite number of data samples ($k = L+1, \ldots, N$), (1) can be rewritten in a matrix form:

$$\underbrace{[\boldsymbol{x}_{L+1} \cdots \boldsymbol{x}_N]}_{\boldsymbol{X}} = \underbrace{[\boldsymbol{h}_L \cdots \boldsymbol{h}_0]}_{\boldsymbol{H}} \underbrace{\begin{bmatrix} s(1) & \cdots & s(N-L) \\ \vdots & \cdots & \vdots \\ s(L+1) & \cdots & s(N) \end{bmatrix}}_{\boldsymbol{S}}. \tag{2}$$

The subspace defined by the rows of $\boldsymbol{S}$ is called the *signal subspace* and is denoted as $\boldsymbol{V}_s$. Its orthogonal complement $\boldsymbol{V}_o$ is referred to as the *null subspace*. When $\boldsymbol{H}$ is of full column rank, $\boldsymbol{X}$ has the same row subspace as $\boldsymbol{V}_s$.

Since the channel matrix $\boldsymbol{H}$ has a dimension of $M \times (L+1)$, it cannot be of full column rank if $M < L+1$. In such a case, we need to smooth the output data vectors so that the rank of the channel matrix can be restored (see [8] for details).

$$\underbrace{\begin{bmatrix} \boldsymbol{x}_{L+1} & \cdots & \boldsymbol{x}_{N-K+1} \\ \vdots & \cdots & \vdots \\ \boldsymbol{x}_{L+K} & \cdots & \boldsymbol{x}_N \end{bmatrix}}_{\boldsymbol{X}_{(K)}} = \underbrace{\begin{bmatrix} \boldsymbol{h}_L & \cdots & \boldsymbol{h}_0 & \cdots & \mathbf{o} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{o} & \cdots & \boldsymbol{h}_L & \cdots & \boldsymbol{h}_0 \end{bmatrix}}_{\boldsymbol{H}_{(K),\text{rblocks}}} \times \underbrace{\begin{bmatrix} s(1) & s(2) & \cdots & s(N-r+1) \\ s(2) & s(3) & \cdots & s(N-r+2) \\ \vdots & \vdots & \cdots & \vdots \\ s(r) & s(r+1) & \cdots & s(N) \end{bmatrix}}_{\boldsymbol{S}_{(r)}} \tag{3}$$

where $r = L + K$, and K is the smoothing factor. As K increases, $\boldsymbol{H}(K)$ will eventually have more rows than columns. The row subspace $\boldsymbol{X}(K)$ becomes the signal subspace if $\boldsymbol{H}(K)$ is of full column rank.

In blind estimation problems, neither $\boldsymbol{S}$ nor $\boldsymbol{H}$ in (2) is known. A further investigation of (3) shows that they both contain rich structure (Hankel) information that can possibly be exploited for blind estimation. In the following sections, we provide an alternate conceptual framework for understanding the deterministic blind symbol estimation problem and demonstrate the feasibility of estimating the inputs directly from the row subspace of $\boldsymbol{X}$.

III. Blind Symbol Estimation

First, we present an important lemma that lays the groundwork for the development of our blind symbol estimation (BSE) algorithm. We define $\boldsymbol{S}(r)$ in (3) the *finite* input matrix of order r. If $r = 1, \boldsymbol{S}(1)$ becomes a row vector, and we refer to $\boldsymbol{s} = \boldsymbol{S}(1)^H$ as the input vector.

Lemma 1: The input vector $\boldsymbol{s}$ can be uniquely determined up to a scalar from the row subspace of $\boldsymbol{S}(r)$ if $\boldsymbol{s}$ has a sufficient number ($> r$) of modes.[1]

Due to space limitation, we cannot provide the proofs of the above lemmas. The complete derivations will be provided in the full version of this paper. Since the row subspace of the input matrix $\boldsymbol{S}(r)$ is often shared by the output data matrix $\boldsymbol{H}(K)$, the physical significance of Lemma 1 lies in the fact that it assures the identifiability of the input sequences from the row subspace of the data matrix.

A. The Proposed Method

Following the assertion in Lemma 1, we derive the BSE method as follows.

Assuming that $\boldsymbol{H}(K)$ is of full column rank, the signal and null subspaces of $\boldsymbol{S}(r)$ can be calculated from the output data matrix $\boldsymbol{X}(K)$. By a property of Hankel matrices, it is shown in [10] that given the null subspace of $\boldsymbol{S}(r)$, the null subspace of $\boldsymbol{S}(r-1)$ can be constructed as $\begin{bmatrix} \boldsymbol{V}_o(r) & \mathbf{o} \\ \mathbf{o} & \boldsymbol{V}_o(r) \end{bmatrix}$, where $\mathbf{o}$ is a $(N - 2r + 1) \times 1$ vector. Following this procedure, we repeat the construction and remove the redundant rows r times, and the null subspace of $\boldsymbol{S}(1)$ has the following form:

$$\boldsymbol{V} = \underbrace{\begin{bmatrix} \boldsymbol{V}_o(r) & \mathbf{o} & \cdots & \mathbf{o} \\ \mathbf{o} & \boldsymbol{V}_o(r) & \mathbf{o} & \vdots \\ \mathbf{o} & \mathbf{o} & \ddots & \mathbf{o} \\ \mathbf{o} & \mathbf{o} & \cdots & \boldsymbol{V}_o(r) \end{bmatrix}}_{r\text{blocks}}. \tag{4}$$

It is readily seen that the input vector $\boldsymbol{s}$ is the unique nontrivial solution of the overdetermined linear equations $\boldsymbol{V}s = \mathbf{o}$.

In practical situations, only noise-corrupted data is available. The inputs can then be estimated by finding the least square solution. The whole algorithm can be summarized as follows:

1) Calculate the null subspace of $\boldsymbol{S}$ from $\boldsymbol{X}$.
2) Construct the $\boldsymbol{V}$ matrix as in (4).
3) Estimate $\boldsymbol{s}$ by solving $\boldsymbol{V}s = \mathbf{o}$.

The strength of the proposed algorithm is its effectiveness in dealing short data sequences, which makes it particularly suitable for wireless systems where the environment changes rapidly. For a large number of data samples, statistics-based methods with better asymptotic efficiency are preferred.

IV. Extension to Handling of Multiple Sources

For $P(P > 1)$ sources, (2) and (3) become $\boldsymbol{X} = \sum_{i=1}^{p} \boldsymbol{H}_i \boldsymbol{S}_i = [\boldsymbol{H}_1 \cdots \boldsymbol{H}_p][\boldsymbol{S}_1^T \cdots \boldsymbol{S}_p^T]^T$. The row subspace of $[\boldsymbol{S}_1^T \cdots \boldsymbol{S}_P^T]^T$ can still be computered from $\boldsymbol{X}$, provided that the channel matrix $[\boldsymbol{H}_1 \boldsymbol{H}_2 \cdots \boldsymbol{H}_P]$ is of full column rank. (To achieve this, smoothing of the output vectors may be required.) This time, however, all the input vectors $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_p$ become the null vector of $\boldsymbol{V}$ in (4). Therefore, if $\boldsymbol{Y}$ is the null subspace of $\boldsymbol{V}$, it is related to $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_p$ by

$$\boldsymbol{Y} = \boldsymbol{W}[\boldsymbol{s}_1 \cdots \boldsymbol{s}_P]^H,$$

where $\boldsymbol{W}$ is a $P \times P$ full-rank matrix.

Without extra information, it does not seem to be possible to remove the ambiguity caused by $\boldsymbol{W}$. Fortunately, for most digital communication signals, the input symbols belong to a finite set of alphabets (e.g., BPSK, QPSK). It was proved in [11] and [7] that blind symbol estimation can be achieved given sufficient data samples. The iterative least squares with projection (ILSP) algorithm introduced in [7] can be used to identify the input symbols from $\boldsymbol{Y}$.

[1] See [3] for the definition of a mode. It can be interpreted an embedded exponential in the input sequence.

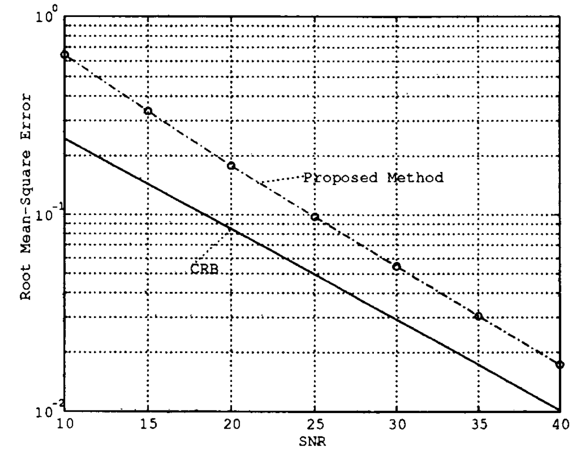

Fig. 1. Root mean-square error versus SNR.

V. Simulation Results

In our simulations, the source symbols were drawn from a QPSK signal constellation. The normalized root-mean square error (RMSE) is employed as the performance measure of the input estimates [8]. The outputs are compared with the Cramér-Rao bound derived in the complete paper version of this letter.

In the first simulation, two antennas were used and the received data were sampled by twice the symbol rate. The effective number of channels is thus 4. The channel responses are described in [3]. Only 20 output vectors were used for symbol estimation. Fig. 1 shows the RMSE of the symbol estimation versus SNR.

In the second simulation, we added a second source. The SNR's for sources 1 and 2 were set at 20 dB and 25 dB, respectively. Forty output vectors were used. Other parameters were identical to that of the first simulation. The combination of the proposed method and the ILSP algorithm performed reasonably well. The average results of 500 trials are summarized in Table I.

TABLE I
MULTIPLE SOURCES SYMBOL ESTIMATION

	SNR	Percent Bit Errors	Iterations
s_1	25	0.100	4.002
s_2	20	0.085	4.002

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