Subcarrier Allocation and Power Control for OFDMA

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Abstract

In this paper, we study the problem of finding an optimal sub-carrier and power allocation strategy for downlink communication to multiple users in an OFDM based wireless system. We formulate the problem of minimizing total power consumption with constraints on BER and transmission rate for users requiring different classes of service, and derive simple algorithms which perform well. We divide larger problem of joint allocation into two steps. In the first step, the number of subcarriers that each user will get is determined based on the users' average SNR. Numerical results demonstrate that the proposed low complexity algorithms offer comparable performance with an existing iterative algorithm [6].

Keywords: OFDM, OFDMA, multiuser, waterfilling, power control.

1 Introduction

One of the biggest advantages of OFDM systems is the ability to allocate power and rate optimally among subcarriers, using "water filling" over the inverse of the channel spectrum. Multiuser power loading and resource allocation strategies allow us to use available resources more efficiently. In this paper we explore this problem in the context of an OFDM based frequency division multiple access (OFDMA) system.

While OFDM based multiple access systems have been proposed, power control and bandwidth allocation for these systems is still largely unexplored. One approach to choosing the subcarrier assignment is to ignore channel information, and simply divide the carriers, in time or frequency, among the users. This approach can be modified somewhat by allowing the number of carriers or the length of the time slot allocated to a user to be proportional to his rate requirement. FDMA, TDMA and CDMA may be used for fixed assignment. OFDM-TDMA allows for greater frequency diversity, and provides a finer granularity in bit rates. Thus, when the subcarrier assignment is not optimized, OFDM-TDMA performs better than OFDMA. Compared to OFDMA with fixed assignment, TDMA is more computationally intensive when optimal bit allocation is used, since each user uses N carriers. In [3] Rohling and Grunheid present a comparison of OFDM-TDMA, OFDMA and MC-CDMA, however the mechanisms of subcarrier allocation for OFDMA is not discussed in detail. In [5], Wahlqvist *et. al.* show that dynamic resource allocation can improve quality of service.

An innovative technique, introduced by Wong *et. al.* [6] applies Lagrangian relaxation to this problem. In Lagrange relaxation, the Lagrange method of optimization is used on an integer parameter, which is "relaxed" to take on non-integer values. Each user is given a power coefficient λ, which determines his "need" for power. This is not a cap on the total power he is allocated, but related more closely to the "water level" in single user water filling. λ has the dual role of regulating both the subcarrier allocation and the total transmission power for each user. The algorithm then iterates, by increasing λ for the user who needs the rate increase the most, reassigning channels and finding the new rates. The reader may refer to [6] for details of the algorithm. Though iterative, this algorithm is guaranteed to converge to a good solution. Unfortunately there are drawbacks to this algorithm, due to the nonlinear nature of the integer problem. The algorithm requires a large number of iterations to converge, and does not converge smoothly.

In this paper we propose a class of computationally inexpensive methods for power allocation and subcarrier assignment. In Section 2, we describe our system model. In Section 3, we explain our approach and describe the algorithms: Bandwidth Assignment Based on SNR, Amplitude Craving Greedy Subcarrier Assignment and Rate Craving Greedy Subcarrier Assignment. The fourth Section presents the simulation studies, and we conclude in Section 5.

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2 System Model and Problem Formulation

The system under consideration is an OFDM system with frequency division multiple access. Perfect channel state information is assumed at both the receiver and the transmitter. Each subcarrier can only be used by one user.

We consider a system with K users, and N subcarriers. Let $r_k(n)$ be the transmission rate for user k on subcarrier n, and $p_k(n)$ be the transmission power for user k on subcarrier n. The two quantities are related by a function $p_k(n) = f(r_k(n))/|H_k(n)|^2$ where $|H_k(n)|^2$ is the channel gain. The rate-power function $f(\cdot)$ depends on the minimum BER requirements, and the available coding and modulation schemes. $R_k = \sum_{n=1}^{N} r_k(n)$ is the total transmission rate for user k. Each user k requires a minimum transmission rate $R_{\min}^k$. We aim to find a subcarrier allocation which allows each user to satisfy its rate requirements while using minimum power:

$$\min \quad \sum_{n=1}^{N} \sum_{k=1}^{K} p_k(n) \qquad (1)$$

$$\text{subject to} \quad \sum_{n=1}^{N} r_k(n) \geq R_{\min}^k, \quad \forall k.$$

3 The Sensible Greedy Approach

In this paper we examine algorithms which fall between these two extremes. Intuitively, we are separating the problem into two stages:

1. *Resource Allocation:* We decide the number of subcarriers each user gets - its bandwidth - based on rate requirements and the users' average channel gain.

2. *Subcarrier Allocation:* We use the result of the resource allocation stage and channel information to allocate the subcarriers to the users.

By solving each step we can find a good, but not necessarily optimal, solution which guarantees a certain level of service. We finish by running the single user power allocation algorithm for each user.

3.1 Resource Allocation Algorithm

In a wireless environment, we expect some users to see a lower overall SNR than other users. These users tend to require the most power. Once each user is given enough subcarriers to satisfy his minimum rate requirements, giving the extra subcarriers to users with lower average SNR will help to reduce the total transmission power. In this section we describe an algorithm which uses the average SNR for each user to decide the number of subcarriers that user will be assigned.

Let H_k be the channel gain of user k on every subcarrier. User k's minimum rate requirement is R_{min}^k. Let user k be allocated m_k subcarriers. Given these subcarriers, to realize his total transmission rate he will transmit R_{min}^k/m_k bits on each subcarrier. Thus his total transmit power will be $m_k f(R_{min}^k/m_k)/H_k$. Our aim is to find a set of m_k, $k = 1, ..., K$ such that

$$\min \quad \sum_{k=1}^{K} \tfrac{m_k}{H_k} f(R_{min}^k/m_k) \qquad (2)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} m_k = N, \quad m_k \in \left\{ \left\lceil \tfrac{R_{min}^k}{R_{max}} \right\rceil, ..., N \right\} \qquad (3)$$

To find the optimal distribution of subcarriers among users given the flat channel assumption, we propose using a greedy descent algorithm similar to that used for discrete water filling.

Algorithm 1 BABS Algorithm

$m_k \leftarrow \left\lceil \frac{R_{min}^k}{R_{max}} \right\rceil, \quad k = 1, .., K.$
while $\sum_{k=0}^{K} m_k > N$ **do**
  $k^* \leftarrow \arg \min_{1 \leq k \leq K} m_k,$
  $m_{k^*} \leftarrow 0,$
end while
while $\sum_{k=0}^{K} m_k < N$**, do**
  $G_k \leftarrow \frac{m_k+1}{H_k} f\left(\frac{R_{min}^k}{(m_k+1)}\right) - \frac{m_k}{H_k} f\left(\frac{R_{min}^k}{m_k}\right), \quad k = 1, .., K$
  $l \leftarrow \arg \min_{1 \leq k \leq K} G_k,$
  $m_l \leftarrow m_l + 1,$
end while

In Appendix A, this algorithm is shown to converge to the optimal distribution of subcarriers among users in flat fading channels, where each user sees the same gain on its subcarriers, if there is enough bandwidth to satisfy the users' rate requirements and $(m_k + 1)f(R_{min}^k/(m_k + 1)) - m_k f(R_{min}^k/m_k)$ is a negative definite, monotonically increasing function of m_k for all users k.

3.2 Subcarrier Assignment Algorithms

Once the number of subcarriers is determined, we move on to assigning specific subcarriers to users. In this section, we propose two suboptimal algorithms to allocate subcarriers to users.

3.2.1 Amplitude Craving Greedy Algorithm (ACG)

If the individual users did not have rate constraints to satisfy we would allocate each subcarrier to the user who has the highest gain on that subcarrier. We use this idea in our next algorithm, but make two modifications: First, each user can only get m_k subcarriers. Once it is allocated m_k subcarriers it cannot bid for any more. Second, the users' average channel gains are normalized to 1, so that users with lower power can have a fair chance when bidding against more powerful users. This procedure is summarized below in Algorithm 2 where $\#C_k$ denotes the cardinality of the set C_k.

Algorithm 2 ACG Algorithm

Ensure: m_k is the number of subcarriers allocated to each user, $C_k \leftarrow \{\}$ for $k = 1, ..., K$.
  for each subcarrier, $n = 1 : N$ **do**
    $k^* \leftarrow \arg\max_{1\le k\le K} |H_k(n)|^2$
    while ($\#C_{k^*} = m_{k^*}$) **do**
      $|H_{k^*}(n)|^2 \leftarrow 0,$
      $k^* \leftarrow \arg\max_{1\le k\le K} |H_k(n)|^2$
    end while
    $C_{k^*} \leftarrow C_{k^*} \cup \{n\}.$
  end for

3.2.2 Rate Craving Greedy Algorithm (RCG)

Although the ACG algorithm works quite well in practice, we can do better. We propose using the bandwidth and power assignments obtained by the BABS algorithm to replace the rate constraint in the original problem, and solve the dual problem:

$$\max_{\rho_k(n)\in\{0,1\}} \sum_{k=1}^{K}\sum_{n=1}^{N} r_k^*(n)\rho_k(n) \tag{4}$$

subject to

$$r_k^*(n) = \begin{cases} 0 & \text{if } f'^{-1}(\lambda_k^*|H_k(n)|^2) < 0, \\ f'^{-1}(\lambda_k^*|H_k(n)|^2) & \text{otherwise.} \end{cases} \tag{5}$$

$$\sum_{n=1}^{N}\rho_k(n) = m_k \quad \forall k, \qquad \sum_{k=1}^{K}\rho_k(n) = 1 \quad \forall n$$

Here, we use the $\lambda_k^* = m_k/H_k f(R_{min}^k/m_k)$, from the BABS algorithm as an estimate of the "water level" for this user after optimal power allocation. The cost function is linear in the variable of interest, $\rho_k(n)$, but the problem is the well known "set partitioning problem" which is combinatorial in nature, i. e. we aim to find a partition, $\mathcal{A}$ of the set of subcarriers, $\{1, ..., N\}$ into K sets, $\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_K$ so as to maximize $\sum_{k=0}^{K-1}\sum_{\{n\in\mathcal{A}_k\}} r_k(n)$. One algorithm to solve this problem may be as follows

- Initialize the partition $\mathcal{A}$ by greedy allocation.
- While there exists some user k such that $\#\mathcal{A}_k > m_k$, remove a subcarrier from user k, and add a subcarrier to a user l such that $\#\mathcal{A}_l < m_l$.

In Appendix B we describe a strategy for the second step which will allow us to solve this problem. However, this strategy is very expensive computationally. Instead we choose to do a nearest neighbors type search, where any user with $\#C_k > m_k$ gives a subcarrier to a user $l^* = \min_{\{l:c_l<m_l\}} \min_{1\le n\le N} r_k(n) - r_l(n)$. We have named this algorithm the Rate Craving Greedy (RCG) Algorithm, since it is initialized using a strictly greedy strategy where the user with the maximum rate is allocated the channel, and all further subcarrier "swaps" are also chosen using a greedy approach. See Algorithm 3.

Algorithm 3 RCG Algorithm

Ensure: m_k is the number of subcarriers allocated to each user, $r_k(n) = f'^{-1}(\lambda_k|H_k(n)|^2)$ is the estimated transmission rate of user k on subcarrier n, $C_k \leftarrow \{\}$ for $k = 1, .., K$.
  for each subcarrier n = 1:N, **do**
    $k^* \leftarrow \arg\max_{1\le k\le K} r_k(n)$
    $C_{k^*} \leftarrow C_{k^*} \cup \{n\}.$
  end for
  for all users k such that $\#C_k > m_k$, **do**
    while $\#C_k > m_k$ **do**
      $l^* \leftarrow \arg\min_{\{l:\#C_l<m_l\}} \min_{1\le n\le N} -r_k(n) + r_l(n)$
      $n^* \leftarrow \arg\min_{1\le n\le N} -r_k(n) + r_{l^*}(n)$
      $C_k \leftarrow C_k/\{n^*\}, C_{l^*} \leftarrow C_{l^*} \cup \{n^*\}.$
    end while
  end for

4 Simulations

The system under consideration has parameters given in Table 1. The channel model is multipath fading with an exponential intensity profile. We assume that 10% of the users will be using video, 40% will be using voice services and the remaining 50% will transmit data. Video and voice traffic require a constant transmission rate of 64kbps and 16kbps respectively, data traffic is assumed to be exponentially distributed, with mean 30kbps. Adaptive modulation is used both with and without power loading of subcarriers in simulations. While power based loading increases the throughput, it is computationally costly and may not be used in a practical system[2].

In simulations we have compared our sensible greedy algorithms, Bandwidth Assignment Based on SNR with Amplitude Craving Greedy subcarrier assignment (BABS-ACG) and BABS with Rate Craving Greedy subcarrier assignment (BABS-RCG), described in section 3 to OFDM-TDMA and the Lagrange Relaxation algorithm (LR) [6]. The relaxation of the $\rho_k(n)$ parameter means that a user

Bandwidth	4 MHz
Number of subcarriers	512
Rate per subcarrier	7.81 KSymbols/sec
Modulation schemes	BPSK, QPSK, 16-,64-QAM
Convolutional Codes	Rate 1/2, 2/3, 3/4

Table 1. System Parameters

which attains its rate by sharing subcarriers may end up with too few when the integer constraint is tightened. The outage probability of the greedy algorithms depend only on the the results of the BABS algorithm. To compare the algorithms, we study the power requirements given the same outage rates (Figures 1 - 2). The LR algorithm was run first, to determine the rates that the algorithm could satisfy. Then TDMA, BABS-ACG and BABS-RCG were run for the same channel conditions, and the same users. While the LR algorithm requires the least power, both BABS-ACG and BABS-RCG do better than OFDM-TDMA.

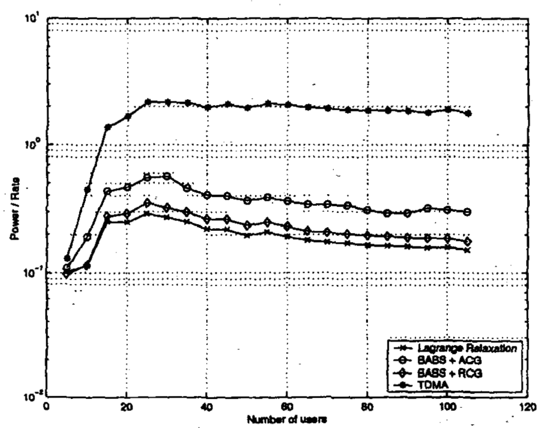

Figure 1. With power based water filling, power per rate vs. user, same outage probability for all methods.

Method	Order of operations
LR	$\mathcal{O}(N \times P_{iter})$, $P_{iter} >> K, N$.
BABS	$\mathcal{O}(K \times N)$
ACG	$\mathcal{O}(K \times N)$
RCG	$\mathcal{O}(K \times N + N \log N)$

Table 2. Algorithmic Complexity.

In Table 2, we compare the worst case performance of each algorithm. P_{iter} represents the number of iterations of the LR algorithm. This number tends to grow larger for more users and uneven average power distributions. Figures

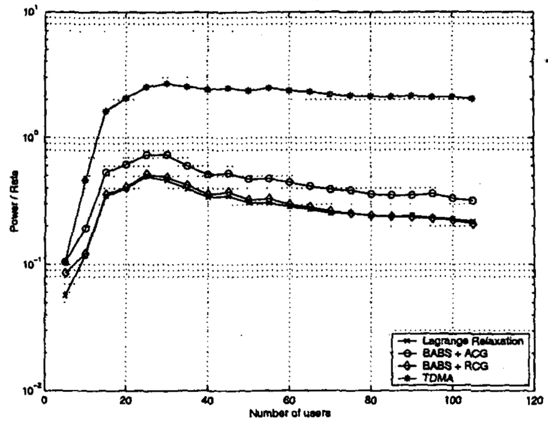

Figure 2. Without water filling, power per rate vs. user, same outage probability for all methods.

3-4 are plots of the average computational complexity found over 300 simulations. We coded the two methods in C, the platform for implementation was a SUN ULTRA-10 machine with 17.9 SPECint_95 and 22.7 SPECfp_95 ratings. Plots of the required CPU time show that both greedy algorithms perform an order of magnitude faster than the iterative LR algorithm, but BABS-ACG performs about twice as fast as BABS-RCG.

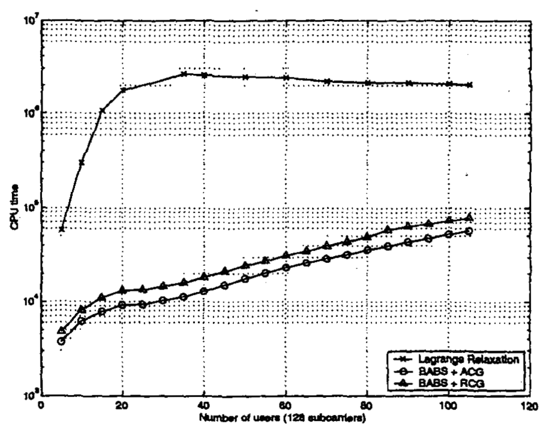

Figure 3. With water filling, average CPU time required vs. number of users (128 carriers).

5 Conclusion

Fast and efficient multiple access algorithms allow mobile networks to adapt quickly to changes in the environ-

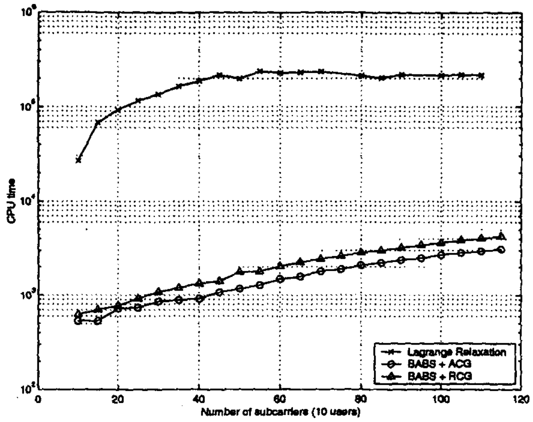

Figure 4. With water filling, average CPU time required vs. number of carriers (10 users).

ment. In this paper, we have described a computationally efficient class of algorithms for allocating subcarriers and power among users in a multicarrier system. By dividing the problem into two stages, we have been able to devise algorithms with low, which operate well under realistic channel and data traffic assumptions. Our approach allows efficient use of system resources, in terms of transmission power, bandwidth efficiency and computational time. Simulation show that the algorithms yield low outage probability, and low power requirements at reasonable complexity, showing that a good resource allocation strategy can be achieved by efficient algorithms in a practical system.

A Proof that the BABS algorithm is optimal

We assume that $(m_k + 1)\ f(R^k_{min}/(m_k + 1)) - m_k f(R^k_{min}/m_k)$ is a negative definite, monotonically increasing function of m_k for all users k.

Let $\{m_k\}$ be the distribution of carriers that our algorithm finds, define $G_k(m) = \frac{m}{H_k} f(R^k_{min}/m)$. Assume that there exists another distribution of carriers among users, $\{n_k\}$ such that $\sum_{k=0}^{K-1} G_k(n_k) < \sum_{k=0}^{K-1} G_k(m_k)$ which differs from $\{m_k\}$ in at least one term. Then there exists at least one p and at least one q such that $n_p > m_p$ and $n_q < m_q$. By assumption $G_q(m_q+1) - G_q(m_q) > G_q(n_q + 1) - G_q(n_q)$ and $G_p(m_p) - G_p(m_p - 1) \leq G_p(n_p) - G_p(n_p - 1)$

Since distribution $\{n_k\}$ is optimal if we reassign one carrier from user p to user q, the total power will increase, i. e. $G_q(n_q + 1) - G_q(n_q) > G_p(n_p) - G_p(n_p - 1)$. Thus we find $G_q(m_q + 1) - G_q(m_q) > G_p(m_p) - G_p(m_p - 1)$. But at some stage in our algorithm, user p was allocated his last carrier, and at that stage, user p had $\tilde{m}_p = m_p$ carriers, user q had $\tilde{m}_q \leq m_q$ carriers. By virtue of the algorithm and the assumption $G_p(m_p) - G_p(m_p - 1) \geq G_q(\tilde{m}_p + 1) - G_q(\tilde{m}_p) \geq G_q(m_q + 1) - G_q(m_q)$. Thus we have a contradiction.

B The Optimal Rate Craving Algorithm

The Rate Craving Greedy algorithm in section 3.2.2 is based on the optimal rate allocation algorithm described below. The proof of optimality is omitted for brevity.

Let $\mathcal{A}$ be a partition of subcarriers $\{1, ..., N\}$ into K sets $\mathcal{A}_1, ..., \mathcal{A}_K$ (an allocation of N subcarriers to K users). We define $\mathcal{G}(\mathcal{A}) = (V, E(\mathcal{A}))$ to be a directed graph with K nodes representing the sets $\mathcal{A}_k$ and $K(K-1)$ vertices. Each edge $e_{k,l}$ represents taking a subcarrier $m = \arg\min_{n \in \mathcal{A}_k} -r_k(n) + r_l(n)$ from node k and reassigning it to node l. Edge $e_{k,l}$ has weight $w(e_{k,l}) = \min_{n \in \mathcal{A}_k} r_k(n) - r_l(n)$.

Theorem: The following algorithm terminates in a partition $\mathcal{A}$ such that each set $\mathcal{A}_k$ contains exactly m_k subcarriers and $\sum_{k=0}^{K-1} \sum_{n \in \mathcal{A}_k} r_k(n)$ is maximized:

1. Initialize the partition $\mathcal{A}$ by assigning subcarrier n to user $k^* = \arg\max_k r_k(n)$.

2. While there exists a node (user) k such that $\#\mathcal{A}_k > m_k$, follow the minimum weight reassignment in $\mathcal{G}(\mathcal{A})$ from node k any node l such that $\#\mathcal{A}_l < m_l$.

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